QUALITY ANALYSIS OF 3D ROAD RECONSTRUCTION

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ABSTRACT:
Quality of 3D reconstructed roads strongly depends on input data and following processing steps. Quality analysis is essential for building up a reliable reconstruction process and for a proper use of 3D data. It is therefore of interest to analyse which error sources influence the final result, and what is the sensitivity of each of these error sources. In this paper we explicitly describe quality of 3D reconstructed roads as a function of input data. These 3D roads have been reconstructed automatically by a fusion process of two input data sets: topographic map data and airborne laser data. Heights of map points are calculated by least squares plane fitting through a selection of neighbouring laser points. We determine the precision of map point heights by using error propagation techniques and properties of least squares adjustment. Map points heights have been calculated with a precision varying from a few centimetres to a few decimetres, depending on the point density and distribution of laser data. Even more important is that independent reference data showed the correctness of predicted quality by testing the actual quality against the predicted quality.

1. INTRODUCTION
Reconstructing 3D topographic objects has been an active research topic in the last decade, driven by the growing need for 3D geo-information and the growing technical possibilities.
Researchers proposed several acquisition techniques varying in terms of level of automation, focus on specific objects, and kinds of data sources like stereo imagery or laser altimetry data. Quality parameters of these 3D reconstructed models strongly depend on input data and how well these objects can be extracted from the data. (Kaarstinen et al, 2005) reviewed the quality of building models submitted by 11 participants, and relate this to the acquisition methods, divided into image based and laser altimetry based approaches. In other individual papers sections on quality assessment are often limited to a value of success rate and completeness or a table of differences between reference data and reconstructed models.

While users of 3D geo-information also gained experiences in their applications, requirements on data quality became more specific. For one purpose users need a higher accuracy than for others. Quality descriptions are therefore essential for a proper use of data. For users as well as for researchers it is of interest to analyse which error sources influence the final result, and what is the sensitivity of each of these error sources.

In this paper we explicitly describe quality as a function of input data, using error propagation techniques and properties of least squares adjustment. Our focus is on the quality of 3D road reconstructions. We will examine the precision of only the height component in these models. Three-dimensional roads are important features for infrastructural analysis, like traffic noise simulations, but are also essential features in 3D city models, besides 3D buildings. Roads can automatically be reconstructed in 3D using airborne laser scanner data in combination with existing 2D map data (Oude Elberink and Vosselman, 2006). Their method recognises and models height discontinuities to allow roads to cross in 3D. Results have been shown for a complex interchange, but quality assessment was limited to a section about completeness of the reconstructed model. First we will generally describe the reconstruction approach, which is an extension to the method of (Oude Elberink and Vosselman, 2006). By using formulas from least squares adjustments and error propagation techniques, we are able to analyse the precision and reliability of our reconstructed model. Finally, we check our reconstructed model by comparing it with independent reference data. Differences between these two datasets should be explainable by the predicted quality measures. Detailed insight in the quality of 3D reconstructed roads is important to analyse critical steps in the reconstruction process. This is especially true in situations in which laser points are scarce like on lower parts at interchanges. This paper gives insight in the quality of the 3D road reconstruction process and results.

2. 3D ROAD RECONSTRUCTION

Essential in our quality analysis is the integration of functional and stochastic information, using error propagation and least squares adjustment techniques. In this section we describe the functional information, covering the subsequent steps to reconstruct 3D roads. Our aim is to reconstruct 3D road models by adding height values from laser data to 2D planar coordinates of map polygons.

2.1 Pre-processing
In a pre-processing stage laser data has been segmented into piecewise smooth laser segments. We have filtered small segments to remove points on objects like cars and traffic signs.

2.2 Assigning laser data to map data
Roughly, our approach assigns laser points to a map polygon and then reconstructs its 3D boundaries by fitting a plane through a selection of the assigned laser points. When reconstructing complex interchanges, assigning laser points to the map needs extra attention. Simple points-in-polygon operations will fail because the existence of roads on multiple height levels. Laser points should be assigned to a road part on the correct height level. In Figure 1 a part of an interchange is shown, visualizing map polygons bounded by black lines, and laser points coloured by height. Colours indicate height above mean sea level varying from yellow (~0 meter), green (~6 meter), blue (~14 meter) and purple (~21 meter).
Looking at a complex infrastructural object like in Figure 1, the following characteristic problems may occur:

P1. Due to a horizontal displacement between map and laser data, laser points will be assigned to the wrong (neighbouring) polygon.

P2. Height data might be acquired at different levels at the same horizontal location because of the across track scanning angle. When reconstructing this map polygon at different height levels, we have to select the right laser points for the right height level, and remove the false laser points.

P3. Problems arise when handling polygons with only a few points, due to the size of the polygon or due to the surface material of the object feature resulting in a low point density.

Problems mentioned above are solved in a special map growing algorithm. Map polygons are merged together if they belong to the same road. Geometric and topological information from two neighbouring polygons decides if they belong to the same road. Geometric and topological information from two neighbouring polygons decides if they belong to the same road. Geometric and topological information from two neighbouring polygons decides if they belong to the same road.

Our plane parameters \( (p) \) can be written in the form:

\[
\hat{p} = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix}^T
\]

By fitting a plane through a selection of laser points within a certain radius, see Figure 2. This plane is calculated by least squares adjustment, these three error sources act as systematic errors, not stochastically influencing the quality of the plane.

Where \( p_1 \) and \( p_2 \) are two slope parameters and \( p_3 \) a distance parameter. We can write the plane calculation in a system of linear equations:

\[
E(y) = Ax.
\]

In equation (2), \( y \) contains observations (z-values of laser points), \( x \) is a vector of the three unknown plane parameters and matrix \( A \) contains information about the configuration of laser points. Each row consists of the horizontal location of a single laser point (\( x, y, 1 \)).

\[
E_1 = \begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} x_3 & y_3 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} x_4 & y_4 & 1 \end{bmatrix}
\]

To solve these equations in a least squares adjustment, observations are given a weight, and plane parameters are estimated by:

\[
\hat{x} = (A^T A)^{-1} A^T y.
\]

After map height calculation, 3D boundaries are triangulated to get a solid surface description of the road. In the next section a quality description is given concerning the height values of 3D roads.

3. QUALITY DESCRIPTION

By using formulas from network design analysis, we can predict the quality of our reconstructed model before the actual reconstruction. For researchers quality prediction is useful for optimizing parameters used in their algorithms ("designing the network"). For users, predicting quality is important because it answers the question whether the input data and the processing steps can fulfill the user requirements.

We distinguish three components in the precision of the map point:

\[
\sigma^2_{\text{map}_\text{out}} = \sigma^2_{\text{block}} + \sigma^2_{\text{error block}} + \sigma^2_{\text{plane out}}
\]

\( \sigma^2_{\text{map}_\text{out}} \) is the uncertainty caused by variations in the plane parameters, which are influenced by laser point noise. \( \sigma^2_{\text{error block}} \) represents a stochastic value for systematic errors in laser data, and \( \sigma^2_{\text{plane out}} \) stands for discrepancies between the fitted plane and the actual shape of the road.

3.1 Quality of plane at map point location

To predict uncertainty in the plane parameters we need information about the quality and configuration of the input data. (Crombaghs et al., 2002) present a practical method to describe quality of laser data sets as a function of four error sources (error 1 to 4, denoted as E1 to E4). These error sources are point noise (E1), GPS (E2) and INS noise (E3) and strip adjustment noise (E4). Influence of each of these error sources depend on the size of the area of interest. Within the radius for selecting laser data, it can be expected that all laser points are influenced by the same E2, E3 and E4. When using least squares adjustment, these three error sources act as systematic errors, not stochastically influencing the quality of the plane.

Our plane parameters \( (p) \) can be written in the form:

\[
z = f(x, y) = -xp_1 - yp_2 + p_3
\]
equation. These error sources will be added later to the precision of the map point (see eq. 9). When only assuming influence of point noise in equation (4), Qₜ turns into a diagonal matrix and (4) can then be written in the form:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{y}.$$  

Equation (6) shows that a diagonal matrix Qₜ does not have an effect on the estimation of plane parameters. However, it does affect the quality of the plane parameters.

$$Q_{\text{block}} = (t_{\text{Q}} t_{\text{A}})^{-1} = \frac{1}{\sigma_z^2} \begin{bmatrix} \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} y_i x_i & \sum_{i=1}^{n} y_i^2 \end{bmatrix}.$$  

In order to avoid singularity when inverting the 3x3 normal matrix, columns of $\mathbf{A}^T \mathbf{A}$ have to be linearly independent. This can be achieved by selecting at least three laser points that do not lie in a straight line. For a stable calculation we proceeded with local coordinates by subtracting the mean location of the laser points. Once the quality of plane parameters is known, we can calculate the height precision of the plane at the location of the map point.

$$\sigma_{\text{plane}}^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2.$$  

3.2 Quality of laser block

Remember that equation (5) consisted of multiple components: plane uncertainty, systematic errors in laser data and model uncertainty. Laser point noise was taken into account in the plane uncertainty; other errors in laser data (E₂, E₃, and E₄ as mentioned in section 3.1) did not reflect the plane equations. However, they influence the precision of the map point height. We can group these errors by

$$\sigma_{\text{block, laser}}^2 = \sigma_{\text{E2}}^2 + \sigma_{\text{E3}}^2 + \sigma_{\text{E4}}^2.$$  

3.3 Quality of plane model

Plane model quality covers the discrepancy at the map point between the actual shape of the road and the modelled plane. If the horizontal distance between map point and laser points is small it can be expected that a plane through these laser points accurately represents the road height at the map point. Model uncertainty becomes of interest when we need to extrapolate over a certain distance, in case we are short of laser points. We can quantify the differences between a local plane and the actual shape, by analysing the curvature of roads. This quantification is a function of horizontal distance between plane origin and map point. To estimate the idealisation precision, we have to use height differences between plane and reality instead of curvatures. For distances smaller than a few hundred meters, we can approximate the difference between the road and a plane by a quadratic term.

![Figure 3. Extrapolation error caused by model uncertainty.](image-url)

Figure 3 can be translated into a stochastic measure for model uncertainty by calculating the standard deviation of extrapolation errors as a function of the distance. We have approximated this value by dividing maximum extrapolation error, calculated by integrating curvatures, by three.

Now that we have described three components that contribute to the uncertainty in map point heights, we analyse the influence of one of the reconstruction parameters—radius—to this uncertainty. Increase of the radius results in the increase of laser points. Generally this will improve the quality of the fitted plane, because the number of observations in the plane calculation increases. However, increasing radius results in larger extrapolation uncertainties. Remember that this extrapolation error increases quadratically with larger radius. The optimum value can be found by minimizing the sum of these two components as a function of the radius value. For practical reasons, our program starts with an initial radius value, which will be increased if there are too few laser points to precisely fit a plane.

4. TESTING WITH REFERENCE DATA

In the previous sections we have described our 3D road reconstruction method and its stochastic model. To be able to test our functional and stochastic model, heights on reconstructed roads have been compared with independent reference data.

4.1 Reference data

Accurate geometric information of highways in the Netherlands is stored in a photogrammetrically derived topographic database, called DTB. Terrestrial measurements have been added to complete road information underneath interchanges and in tunnels. The DTB contains 3D geometric and semantic information of points, boundaries, centrelines and surface features of national roads, at a map scale of 1:1000. This also includes information on road details like locations of paint strips, traffic lights, road signs and other detailed infrastructural objects. DTM information (2.5D) has been integrated into the DTB by photogrammetric measurements on breaklines in the terrain. An example of DTB data is given, showing a complex interchange Prins Clausplein near The Hague.

![Figure 4. DTB data is used for reference information. Paint strips, shown as blue lines, have been selected to test reconstructed roads.](image-url)
4.2 Quality of 3D roads by comparing to reference data

In this section we will describe our testing configuration by comparing reference data with our reconstructed model. As we have seen in section 2 roads are represented as a TIN surface, using 3D map points on the boundary as TIN nodes. Figure 5 explains the set-up of our height testing procedure. Orange bullets represent 3D positions on paint strips, which are measured with high accuracy in the reference dataset. At these green plus marks height differences have been calculated.

Our expectation is that the height difference between reference data and our 3D model should vary around zero. Deviations should be explainable by uncertainty in the 3D model and in the reference data.

\[ \Delta h_i = h_{ref, i} - h_{mdl, i}, \sim N(0, \sigma_{\Delta h_i}) \]  

(10)

The term \( \sigma_{\Delta h_i} \) contains the height variance of the model, at the location of the reference point. We therefore have to propagate precisions of the map points, calculated as described in section 3.2, to the location of the reference point. Looking again at Figure 5, we see that the precision of three map points influence the precision at reference point location.

First, the location of the reference point within the TIN mesh is important to describe the influence each of the map points. If the reference point is close to one of the three map points, the precision of the TIN height is highly influenced by the precision of the height of this single map point.

Then we investigate the influence of covariance between the three map points. Extreme cases here are no covariance and full covariance. If the three map point heights have been calculated by three different groups of laser points, we can assume that the correlation equals zero. This occurs when using a small radius to select laser points. If the three map point heights have been calculated by the same group of laser points, the correlation equals one.

\[ \sigma_{\Delta h_i} = \frac{\sigma_{\Delta h, TIN}}{\sigma_{\Delta h, TIN}^2 + \sigma_{\Delta h, TIN}^2} \]  

(11)

Equation shows the calculation of the precision of the reconstructed model, at locations of reference points, by TIN interpolation of 3 precision values of three map points, divided by a correlation term \( \alpha (1 < \alpha < \sqrt{3}) \).

4.3 Testing our predicted quality

In section 3 we have calculated the precision of map point heights by using error propagation techniques and properties of least squares plane fitting, in this section followed by an actual quality check using reference data. To test the stochastic model we check if the actual differences can be explained by the predicted accuracy. With the outcomes of equation (10), we test if the difference is significant by using a modified version of the w-test statistics on local error detection as described by (Baarda, 1968 and Teunissen, 1991). In their approaches, the w-test calculates normalized residuals of geodetic observations. If the test exceeds a critical value, this observation will be recognized as a possible outlier. In an iterative procedure the observation with the highest w-test value has been removed from the adjustment.

\[ w_i = \frac{\Delta h_i}{\sigma_{\Delta h_i}} \sim N(0,1) \]  

(12)

A closer look at the \( w_i \) learns that it indicates how well one can predict the actual quality. This is an informative measure to show if the predicted quality represent the actual quality. If the stochastic model is correct, the total of all w-test values should have a standard normal distribution. To rely on predicted quality is important for future users who want to predict the quality of 3D reconstructed roads, without checking on highly detailed reference data. Besides this, reference data might not be available at some locations. Large w-test values indicate that the actual quality is worse than predicted. In our approach it is of interest to find reasons for large w-test values, because the functional or stochastic model might not be correct at those locations.

5. RESULTS

5.1 Data specifications

For a complex interchange we assigned coarse laser data (~1 point/9m) to a medium scale topographic map (1:10.000). Laser point noise has been determined in a quality control procedure at the Survey Department of Rijkswaterstaat. For this project the laser point noise (E1) has been stated to be 8 cm, GPS noise (E2) 3 cm, INS noise (E3) 4 cm and block uncertainty (E4) 3 cm. To estimate extrapolation errors due to model uncertainty, we analysed curvature of road heights. Maximum slope differences on highways can be found near interchanges, hillsides and exits. Terrestrial measurements show that slope differences at such locations are about 2% per 100 meter.

\[ \Delta y(d) = 2 \cdot 10^{-4} d \]  

(13)

We can derive the formula for maximum height difference as a function of distance by integrating formula (13).

\[ \Delta y(d) = 1 \cdot 10^{-4} d^2 \]  

(14)

And its standard deviation:

\[ \sigma_{\Delta y, sat} = (1 \cdot 10^{-4} d^2)^{1/2} \]  

(15)

Theoretically, we have to optimize the radius for each map point, because of varying laser point configuration and (thus) plane uncertainty. Instead, we decided to use a default radius of 15 meter, which will be doubled in case less than three laser points are found in this radius.

5.2 Predicted standard deviation of map point heights

Figure 6 shows predicted standard deviations of map point heights. The figure shows the position of map points, coloured by predicted standard deviation of the map point height.
visibility reasons the standard deviation has been classified into three categories: standard deviations larger than 50 cm (shown in red), larger than 20 cm (yellow), and below 20 cm (green). To better understand the cause of large variations at some locations, the blue box in Figure 6 shows the laser points used for 3D road reconstruction. The relation between lack of laser data and large height variations can easily be seen for locations in black ellipses. Point densities in these black ellipses drop to 1 point per 100 m², with extremes to 1 point per 600 m². At map point locations in those areas, map point heights show standard deviations of more than 50 cm. Two factors play an important role here. First, the plane has been determined by just a few laser points; standard deviations of laser points will have a great influence because they are not averaged out. Secondly, the search radius for finding enough laser points increases up to 50 or even 100 m. This results in extrapolation errors rising up to 50 cm or more.

Figure 6. Standard deviations of map point heights. Compare with available laser points (lower right corner).

Bad configuration of the laser points leads to large standard deviations. Figure 7 shows a situation where the majority of laser points lie on a straight line, in this case clearly measured in just one or two scan lines. Fitted planes are badly determined in the direction perpendicular to this scan line. Blue circles have a radius of 15 meter.

Figure 7. Bad configuration of laser points (left) leads to large standard deviations (right).

To better visualize the results, Figure 9 shows the height difference between reconstructed model and reference data. A further look at Figure 9 learns that in the centre of the interchange (highlighted in the lower right corner box), where laser points were scarce at all height levels, the calculated differences are remarkably small. A few differences are more than 50 cm, some below 50 cm and many below 20 cm (green). In the lower left corner box, two situations are highlighted which show large height differences with a systematic character. In the higher circle height differences could be expected, due to the lack of laser points, see Figure 6. The reason for differences in the lower circle is that the search radius selects laser points from both road parts, which happen to curve strongly at those locations. Therefore, fitting a plane through the selected points will differ from reality.

Number of reference points inside test area 10922
Mean difference 0.5 cm
Standard deviation of vector of differences 15.4 cm
Maximum absolute difference 121 cm

Table 1. Statistical results of comparing heights of 3D roads.

Table 1 summarizes most important statistic information of height differences between reference data and 3D reconstructed model. The mean difference includes systematic errors between reference data and our reconstructed model. Normally, it is expected to be in the order of 0.5 cm, due to systematic errors in laser data (Crombaghs et al., 2002). In this case, the mean difference happens to be very small (0.5 cm). Looking at the standard deviation of the differences of 15.4 cm, and knowing that it includes uncertainty in the reference data \( \sigma_{\text{ref}} = 9 \) cm, we can calculate the uncertainty of our reconstructed model \( \sigma_{\text{mod}} = \sqrt{15.4^2 - 9^2} = 12.5 \) cm. It should be noted that this value is biased by some systematic errors in the reconstructed model.
5.4 Testing predicted standard deviations

Now that the actual difference is known, we divide each difference with the expected standard deviation of the difference. In Figure 10 large w-test values have been coloured yellow (larger than 3) and red (larger than 4). At these points the actual height difference was three or four times larger than expected, meaning that either the standard deviation was too small or the calculated height was significantly wrong. Note that the former case deals with the stochastic model, and the latter case with the functional model. Due to the systematic character of large w-test values, we assume a functional error causes the problems at those locations, mostly where one road splits into two roads. However, the distribution of all w-tests is close to the standard normal distribution, as 68% of the w-test values are less than 1 and 92% are less than 2. If we remove outliers, standard deviation is 1.06 (with outliers 1.22). This means that the predicted stochastic model is a bit too optimistic, but still realistic.

Independent reference data has been used to test our reconstructed model and its derived quality parameters. Predicted standard deviations realistically represent the actual quality for most of the situations. Exceptions are found at road splitting situations, where actual differences are more than four times higher than expected. The reason is the wrong assumption that a least squares fitted plane through the selected laser points, realistically represent the shape of the road. Future work will focus on improvement of reconstruction of these splitting roads. This can be achieved by selecting only those laser points that lie on the front side of the map point. The search algorithm for laser points should therefore not cross the polygon border.

Quality analysis as presented in this paper is not limited to 3D road reconstruction, but can be extended to other reconstruction applications. For example, building reconstruction can benefit from quality measures by error propagation. Decisions on conflicts between building knowledge and data driven information can be made more reliable if data driven approaches come with quality measures.

6. DISCUSSION

In this paper we have described a method to calculate quality of 3D reconstructed roads by error propagation. These 3D reconstructed models have automatically been acquired by a fusion process of map data and airborne laser data. After assigning laser data to map polygons, heights of map points have been calculated by least squares plane fitting through a selection of laser points inside the polygon. These 3D map points are nodes in the 3D boundary description. Precision of the map point have been calculated by error propagation of laser point noise and the configuration of the laser points used for plane fitting. Also, influences of model uncertainty have been taken into account. Average predicted standard deviation of map point heights is about 10 cm.

Our method combines a 2D topographic data set with an airborne laser scanner dataset (2.5-3D). Even at locations where no height information is available, our method can reconstruct 3D roads with a height precision in the order of 10-15 cm. Input data sets used in this paper are parts of national databases. Now that we can predict quality of 3D roads, we can predict the height quality for all roads in the national database without actually having to reconstruct them, and without testing them with reference data.

ACKNOWLEDGEMENT

We would like to thank the Dutch research programme Space for Geo-Information for funding this research, and Survey Department of Rijkswaterstaat, Topographic Service as well as the Steering Committee ABN for providing the data.

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