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ADJUSTMENT OF LARGE PHOTOGRAMMETRIC BLOCKS

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The solution of normal equations according to the classical elimination method involves considerable computer costs when the number of unknowns increases greatly. This is the case e.g. in photogrammetric block adjustments according to the least square method, with the image coordinates of the points as observations and the six orientation elements of each picture and the ground coordinates of new points as final parameters.

A method both economic and fitted for the computation of large blocks has been taken into use in the National Board of Survey of Finland. The solution of normal equations is based on the block successive overrelaxation iterative method:

$$x_{i+1} = w(x_{i+1,S} - x_i) + x_i \qquad (1)$$

where x is an approximation of the solution vector, x_S the value calculated by the block Gauss-Seidel method, and w the acceleration parameter.

In the diploma work, the effect of w on the speed of convergence, the eigen values of the iteration matrix and the possibilities of choosing the optimum value w_b of the w during the computation are dealt with. Furthermore, the dependence of the w_b of the geometric structure of the block has been studied.

When the value $w \leqq w_b$ is used for the parameter, the eigen values of the iteration matrix are real. Then

- the ratio of the norms of the correction vectors (the norm defined here as the maximum absolute value of the vector elements) of two successive iteration computation cycles converges to the eigen value, ℓ, having the maximum absolute value; this reliable value as a limit is in practice achieved after 20-40 iteration cycles,
- respectively the error norm of the solution decreases linearly with ℓ; thus the residual error of the solution can be estimated by means of the norm of the correction vector,
- w_b can be computed by using ℓ acc. to the following closed formula

$$w_b = \frac{2}{1+\sqrt{1-\frac{(\ell + w - 1)^2}{\ell w^2}}} \qquad (2)$$

When a value w larger than the optimum value is used the eigen values of the iteration matrix are complex. Then:

- the absolute value of each eigen value is equal to ℓ,
- the eigen value needed for the computation of the w_b is obviously not readily computable,
- the speed of convergence is the same for all blocks,
- the components of the correction vector of the solution diminish in magnitude similarly to the amplitude of decreasing oscillation.

The speed of convergence (w_b) obviously depends on the geometric structure of the block in such a way that the convergence in strips is considerably slower than in blocks. The number and location of ground control points have a very remarkable influence on the speed. A strip with few ground control points requires over 20 iteration cycles for one correct decimal of the solution. For a block containing few ground control points the corresponding number of iterations is about 10 , and for a block containing a relatively large number of ground control points,even five cycles are enough.

Though very large blocks have not been computed so far it is evident that the size of the block, the number of new points per picture and the percentage of the overlap have no significant influence on the convergence.

The block adjustment method has been programmed for the IBM S/360/50 computer with a 256 K memory and disk storage IBM 2314. In the programming, special attention has been paid to essential respects affecting the computing time, hereby aiming at a maximum speed. The solution of the normal equations has been programmed in such a way that the w-value will be automatically corrected to the optimum, provided that the initial approximation is below the optimum and that a sufficient number of iterations are computed.

The computer processing time is theoretically and empirically approximately linearly dependent on the number of pictures, on the average number of new points per picture and on the number of iterations needed in the computation. As an additional advantage, this ènables the solution of the normal equations up to a certain number of decimals. The desired accuracy may be an input value for the computation. The maximum capacity of the program is 160 pictures and 750 new points. This capacity may as necessary, be increased by using auxiliary memory facilities without substantially increasing the costs.

The costs involved by the method are moderate and they depend on the accuracy of the initial approximations and on the number of points per picture.

Only small alterations in the unit price, i.e. the price per a new ground control point, can be stated in the case of larger blocks. A disadvantage of the method described above is that the standard errors of the adjusted parameters cannot be computed. Estimates about the accuracy of the adjustment are the residuals of the adjusted observations and the standard error of unit weight.