

## GPS for Photogrammetry

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### *Abstract*

*A number of GPS supported aerial triangulation projects are presented, based on the method of combined block adjustment with linear GPS drift correction parameters. The results prove the high accuracy performance and confirm the ready operational status of the method. Economic and technical application is feasible over the full range of aerial photogrammetry projects.*

### **1. Introduction - Farewell to Einari Kilpelä**

This paper has been presented at the Colloquium of April 1st, 1993, held in honour of Einäri Kilpelä who had decided to leave photogrammetry. At that occasion his colleagues and friends bade farewell, in the appropriate traditional way of a scientific colloquium, to a most respected and reputed teacher and professor who has successfully promoted and represented Finnish photogrammetry in the remarkable continuation of a long tradition with outstanding personalities and exceptional highlights. I have been very pleased to take part in the colloquium, to express my feelings of deep personal esteem and great appreciation to a dear friend with whom I have enjoyed professional and personal contacts for more than 20 years.

As it happens, this paper on the application of GPS to aerial triangulation recalls my first lectures at Helsinki Technical University around 1970 on aerial triangulation and block adjustment especially also with statorscope auxiliary data, which then caught the particular interest of K. G. Löfström. GPS for aerial triangulation completes, in a way, the combined block adjustment with auxiliary camera orientation data which had started at that time.

The topic of this paper is GPS supported aerial triangulation. That topic has been elaborated in publications, and I have given lectures about it, also in Finland. Not wanting to repeat myself reference is made to previous publications. In this paper the emphasis will be on the operational performance and on experience with practical application of GPS supported aerial triangulation.

### **2. GPS-Positioning**

Let us very briefly recall the elementary principles of the NAVSTAR Global Positioning System (GPS). The GPS satellites, orbiting at 20000 km altitude, permanently emit electromagnetic signals for positioning and navigation anywhere on earth at any time. The L1 and L2 carrier waves of 19 cm resp. 24 cm wavelength carry the P-code and C/A-code modulations which are intended for measuring pseudo-ranges in real time. High precision geodetic and photogrammetric positioning makes use of the phase observations of the carrier waves

for ranging. They are precise to 2 mm or better but have the problem of ambiguities. The inherent problem of large errors is greatly reduced by applying differential methods, making use of stationary and roaming receivers simultaneously. For high precision application in geodesy highly sophisticated methods have been developed.

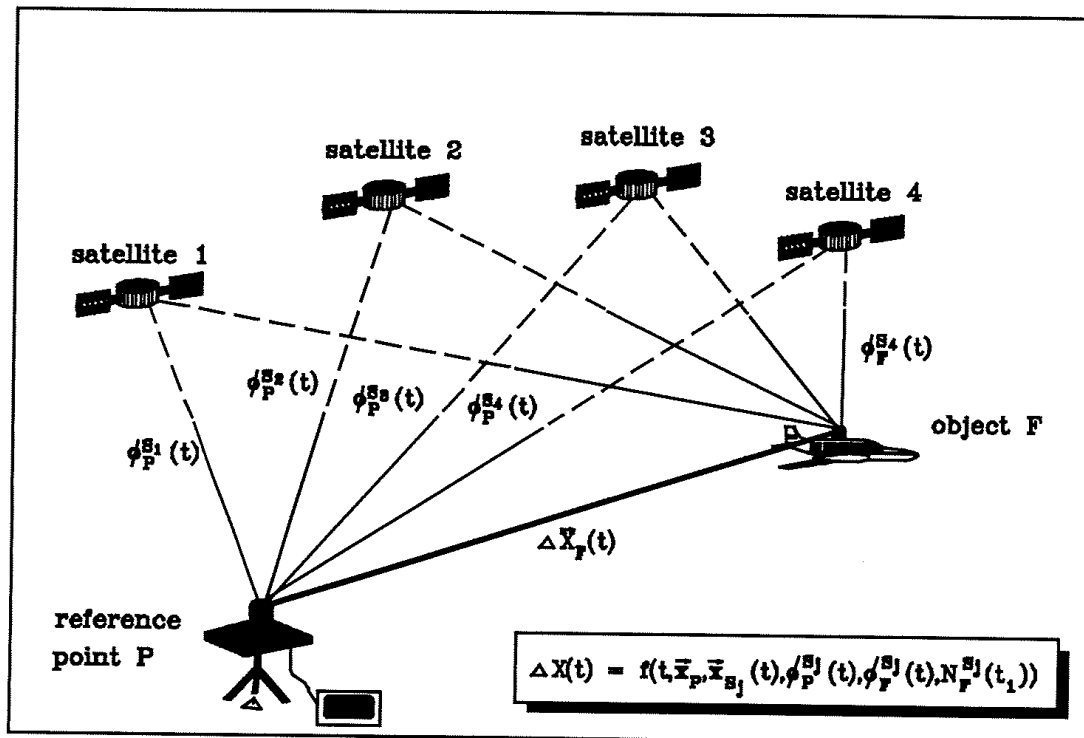


Fig. 1: Relative kinematic positioning

In photogrammetry absolute or relative pseudorange positioning in real time is applied with great benefit for air survey flight navigation. The second major GPS application concerns the precise camera positioning, in connection with aerial triangulation. It concerns relative kinematic positioning on the basis of differential carrier wave phase observations. Unfortunately, the circumstances of air survey operations are unfavourable for airborne GPS positioning of high accuracy. They have prevented, so far, sophisticated error modelling. Air survey flight missions can take several hours of time and move far away from a stationary receiver, up to hundreds of km. Until recently only the use of one stationary receiver has been considered, and only single frequency (L1) receivers have been available or applied. The major problem, however, has been the fact that it is practically impossible to avoid signal interruption and loss of lock when flying turns in operational air survey missions.

In view of those conditions it has been our Stuttgart approach to accept signal disruptions, i.e. not to make special efforts to avoid them by flying low banking angles, to abandon a stationary ambiguity solution before takeoff, to accept single frequency receivers, and to be satisfied with one stationary receiver, possibly located at a great distance from the mission area. The ambiguity solutions are determined repeatedly, per photo strip if necessary, based on C/A-code pseudorange positioning, which may be subject to selective availability (SA). The philosophy of that approach has been explained in former papers. The dominant considerations were economic and convenient flying operations and application of then available

hardware equipment. Most considerations are still valid today.

The essential consequence of that approach is that the ambiguity solutions are biased resulting, with other error effects, in systematic (drift-) errors of the derived GPS positions. Fortunately, they are constant or linear for quite same range. It is left to the combined block adjustment to assess the linear GPS drift errors and to compensate them by linear correction terms.

Some other practical problems of airborne GPS have been sufficiently solved, like the determination of the spatial offset between GPS antenna and the camera in the aeroplane, and the application of the attitude-dependant offset corrections during the block adjustment. The same holds for the time offset between the GPS recordings and the actual exposure of the camera. A problem of more general nature is the GPS datum, resp. the photo-block datum. With biased ambiguity solutions and other error effects the derived GPS positions are not absolute with regard to the WGS 84 coordinate system. Also, a transformation into a national or local reference system of the block has to be provided in most applications. Such datum transformations are included in the block adjustment. They are conveniently based on some ground control points. Thus, in general, the conditions for applying GPS to aerial triangulation are set in realistic operational terms.

The GPS data processing provides preliminary photo-station coordinates, with regard to WGS 84 or an initial transformation into a national system. The GPS data are discontinuous and contain systematic or datum errors.

### 3. Combined Block-Adjustment

#### 3.1. Method

The preliminary GPS camera position data are used for aerial triangulation in the same way as auxiliary data have been used formerly. The GPS coordinates are introduced as weighted observations into the combined block adjustment via additional observation equations. They relate the GPS camera station coordinates directly with the unknown perspective centre coordinates of the block adjustment. The photogrammetric side of aerial triangulation, i.e. image coordinate observations and adjustment approach, is not changed at all, as also a few ground control points are implied. The GPS observation equations are given linear correction terms in order to cope with the GPS drift, resp. datum errors. The correction terms are normally set per strip. The  $3 \times 2$  parameters are treated as unknowns and solved in the block adjustment.

The combined adjustment requires some extension of the conventional block adjustment programs, but does not present any problems, as such. There is, however, the danger of singularities or near singularities. If unknown drift parameters are introduced per strip and only 4 control points used, the block adjustment solution is not sta-

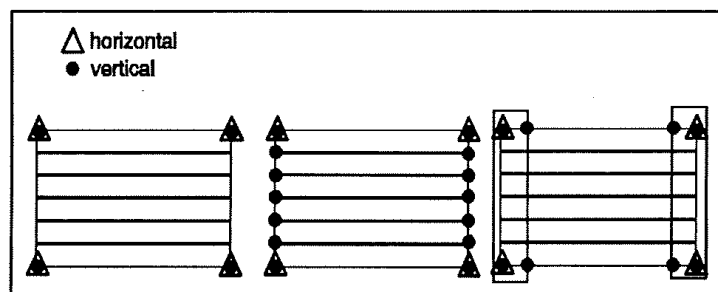


Fig. 2: Control scenario for GPS blocks

ble, in case of standard photo-overlap. There are 3 ways to stabilize the blocks: The use of double coverage (e.g. 60% side overlap), the use of 2 chains of vertical control points, or flying 2 cross-strips, see Fig. 2. The latter scenario, 2 cross-strips with 4 control points in the corners of the (regular) block, is considered the standard case and is generally recommended for application at present.

### 3.2. Accuracy

The introduction of GPS camera station coordinates into the block adjustment has greatly stabilizing effects. As the blocks are essentially attached to the GPS data any major propagation of errors resp. block-deformation is effectively suppressed, which formerly had to be controlled by ground control points. For that reason the number of ground control points can be greatly reduced in GPS blocks, their function being reduced to providing the datum transformation.

GPS data accomplish quite uniform accuracy distribution within a block, and there is little dependence on block-size. The GPS positioning being in the order of 10 cm or better the accuracy of the adjusted blocks is essentially determined by the photogrammetric measuring accuracy of image coordinates ( $\sigma_0 \rightarrow \bar{\sigma}_0$  projected into terrain units). Thus, it is always a question of high precision aerial triangulation. Extended theoretical studies have given insight into all essential accuracy features of GPS blocks, which can be summarized in simple rules. The standard schematic GPS block (2 cross-strips, 4 control points, 6 drift parameters per strip) is generally expected to give a horizontal accuracy (x,y) of  $1.5 \bar{\sigma}_0$  resp. a vertical accuracy (z) of  $2.0 \bar{\sigma}_0$ , as long as  $\sigma_{GPS} \leq \bar{\sigma}_0$ , which is not always the case with large photo scales.

## 4. Practical Application - Empirical Results

### 4.1. Overview

In the following a number of practical projects of GPS supported aerial triangulation are reported and commented. Almost all blocks refer to practical application. Various photogrammetric companies have carried out the GPS-photo-flights and provided the aerial triangulation observations. The INPHO company in Stuttgart has carried out the GPS data processing and the combined block adjustment. The software program SKIP for GPS data processing and the bundle-adjustment program PAT-B, which has been extended to include GPS camera station coordinates and unknown GPS drift-correction parameters, have been used. All cases refer to single frequency (L1) receivers, to only 1 stationary receiver, to blocks flown with at least 2 cross-strips (or additional control, in some cases) and to adjustment with unknown GPS drift parameters applied per strip. All blocks have been successfully adjusted, there were no failures nor serious problems. In that sense it can be claimed that the method is truly operational in practice, having outgrown the initial experimental stage.

In table 1 and table 2 the projects are listed which have been handled by INPHO company between summer 1991 and fall 1993. The list shows that the blocks cover a wide range of application in various countries, of photo scales between 1:4000 and 1:50000, and a blocksize

		year	photo scale	photos	strips	control points	block size
1	Guinea	1991	1:30000	346	15 (11+4)	29	45 x 90 km
2	Germany I	1991	1:8000	90	7 (5+2)	4	6 x 8 km
3	Germany II	1992	1:7500	50	6 (4+2)	4	5 x 6 km
4	Germany III	1992	1:7500	70	8 (6+2)	4	7 x 7 km
5	Germany IV	1992	1:7500	55	6 (4+2)	4	6 x 5,5 km
6	USA I	1992	1:8700	415	17 (14+3)	12	16 x 25 km
7	Germany V	1992	1:6200	1633	45 (39+6)	34	23 x 33 km
8	USA II	1992	1:42000	78	6	4	35 x 35 km
9	USA III	1992	1:34000	106	7	4	35 x 40 km
10	USA IV	1992	1:41000	65	5	4	35 x 30 km
11	Canada I	1992	1:6100	159	10 (8+2)	12	7,6 x 7,6 km
12	UAE I	1992	1:28000	249	12 (10+2)	38	38 x 25 km
13	Germany VI	92/93	1:4000/5000	44	7 (4+3)	4	2 x 2 km
14	Nepal	92/93	1:30000	(3000)	-	-	-
15	USA V	1993	1:14000	183	11 (9+2)	4	20 x 20 km
16	Middle East I	1993	1:50000	143	7 (4+3)	6	133 x 33 km
17	Middle East II	1993	1: 50000	136	7 (4+3)	6	130 x 33 km
18	USA VI	1993	1: 24000	86	8 (5+3)	6	35 x 21 km
19	USA VII	1993	1: 38000	185	10 (6+4)	8	150 x 30 km
20	Canada II	1993	1: 31000	57	5 (4+1)	6	45 x 12 km
21	New Zealand I	1993	1:22000	72	7 (5+2)	4	19 x 17 km
22	UAE II	1993	1: 28000	64	6 (4+2)	4	26 x 20 km
23	UAE III	1993	1: 27000	16	3	7	13 x 10 km

Table 1: Parameters of some GPS-supported aerotriangulation projects

	project	max. flight time per strip [min.]	GPS receiver	data rate	overlap	$\sigma_0$ [ $\mu\text{m}$ ]	theor. accuracy (all block points) [in $\sigma_0$ ] plan./height	absolute accuracy on check points [in $\sigma_0$ ] plan./height
1	Guinea	2 - 16,5	ASHTECH	1,0	60/20	9,0	1,2/2,0	/
2	Germany I	1 - 2	ASHTECH	0,5	60/20	6,3	/	1,3/2,2
3	Germany II	1 - 2	SERCEL	0,6	60/20	4,8	1,3/2,5	1,8/2,3
4	Germany III	1 - 2	SERCEL	0,6	60/20	4,5	1,2/2,6	1,6/2,8
5	Germany IV	1 - 2	SERCEL	0,6	60/20	4,0	1,2/2,6	1,6/2,6
6	USA I	2 - 8,2	TRIMBLE	1,0	60/20	6,4	1,1/1,8	1,8/2,6
7	Germany V	1 - 8,5	ASHTECH	1,0	60/20	7,8	1,3/3,0	/
8	USA II	7,5	ASHTECH	1,0	60/20	7,5	1,2/2,3	/
9	USA III	7,5	ASHTECH	1,0	60/20	8,0	1,6/2,2	/
10	USA IV	7,5	ASHTECH	1,0	60/20	8,0	1,2/2,7	/
11	Canada I	1,6	ASHTECH	1,0	60/20	6,9	0,9/1,6	1,8/1,9
12	UAE I	3,3	ASHTECH	1,0	60/20	9,1	0,9/1,8	/
13	Germany VI	0,7	ASHTECH	0,5	60/60	4,8	0,7/1,4	0,44/1,8
14	Nepal		SERCEL	0,6	60/20	/	/	/
15	USA V	4,1	TRIMBLE	1,0	60/20	6,1	1,2/2,3	1,5/2,4
16	Middle East I	17,5	ASHTECH	0,5	60/20	6,4	0,8/1,4	1,1/1,3
17	Middle East II	16,7	ASHTECH	0,5	60/20	7,0	0,8/1,6	1,5/1,9
18	USA VI	9,3	TRIMBLE	0,5	60/20	6,1	1,1/2,1	/
19	USA VII	47,6	TRIMBLE	0,5	60/20	6,0	1,1/1,9	/
20	Canada II	9,1	TRIMBLE	1,0	60/20	7,1	1,7/3,5	/
21	New Zealand I	6,1	TRIMBLE	0,5	60/20	5,9	1,3/2,2	/
22	UAE II	5,0	SERCEL	0,6	60/20	8,6	1,8/2,6	/
23	UAE III	1,8	ASHTECH	0,5	60/20	5,6	1,6/3,5	/

Table 2: Parameters and accuracy results of some GPS-supported aerotriangulation projects

from 1 strip of 12 photos to a large block of 1633 photos. Different types of GPS receivers have been applied, with data rates of 0.5 sec, 0.6 sec and 1 sec. The photo overlap was standard (nominally 60% forward-, 20% side-overlap) with 1 exception (block 13, 60% side-overlap). The aerial triangulation observations (image coordinates) are generally of good quality, as the  $\sigma_0$ -estimates from the block adjustment indicate.  $\sigma_0$  values between 4.0  $\mu\text{m}$  and 4.8  $\mu\text{m}$  (blocks 3, 4, 5, 13) refer to blocks with signalized points. All other blocks ( $\sigma_0$  between 5.6  $\mu\text{m}$  and 9.1  $\mu\text{m}$ ) had artificially marked or natural tie points.

Table 2 shows some accuracy results of the adjusted GPS-blocks, usually based on an assumed a priori precision of GPS camera station coordinates of 10 cm. For most blocks the theoretical r.m.s. accuracies of all block points are given, expressed in units of  $\sigma_0$ . They have been derived by the inversion of the normal equation coefficient matrix. The average results, excluding block 13 with 60% side-overlap, amount to 1.2  $\bar{\sigma}_0$  in x,y, and 2.3  $\bar{\sigma}_0$  in z, which is reasonably close to the generalized expectation of 1.5  $\bar{\sigma}_0$  resp. 2.0  $\bar{\sigma}_0$ . The range of accuracy (0.8 - 1.8  $\bar{\sigma}_0$  in x,y, 1.4 - 3.5  $\bar{\sigma}_0$  in z) indicates however, that the different block configurations and different cross-strip and ground control scenarios cause variations against the schematic theoretical case. The exceptionally good results of blocks 11, 12, 16, 17, 18, 19 refer to blocks with either 3 or 4 cross-strips or increased numbers of ground control points. In 8 of the blocks (again excluding block 13) some check points were available for independent assessment of the absolute accuracy of the adjusted blocks. The average r.m.s. accuracy results, as derived from check points and quoted in table 2 amount to 1.6  $\bar{\sigma}_0$  in x,y and to 2.2  $\bar{\sigma}_0$  in z. They are to be compared with the average theoretical accuracy of the same blocks, being 1.1  $\bar{\sigma}_0$  in x,y and 2.1  $\bar{\sigma}_0$  in z. Obviously the theoretical horizontal accuracy expectations are somewhat too optimistic, whilst empirical and theoretical accuracy agree very well in z. Besides, the empirical accuracy results are quite close to the general schematic expectation of 1.5  $\bar{\sigma}_0$  resp. 2.0  $\bar{\sigma}_0$ . The overall importance of the empirical accuracy results is, however, the confirmation that in all cases high accuracy is obtained relative to the photo scale. The high accuracy performance of GPS-blocks is consistent in terms of image coordinate precision and nearly independent of circumstantial parameters. In 1:10000 photo scale, for instance, with  $\sigma_0 = 7 \mu\text{m}$  image coordinate precision, GPS blocks are expected to have 11 cm horizontal accuracy and 15 cm or 0.1‰h vertical accuracy. Such accuracy results would be obtained in conventional aerial triangulation, i.e. without GPS, only with a considerable number of control points.

#### 4.2. Various block configurations

The schematic scenario of GPS supported blocks, according to fig. 2, concerns more or less rectangular blocks, with 2 cross-strips and 1 control point in each corner of the block. Such standard cases did occur in the blocks 2, 3, 4, 5, 21, 22, see example of fig 3. Their empirical and theoretical accuracy results agree reasonable well with the general accuracy prediction. A variation shows block 11, which has clusters of 3 control points in each corner of the block (fig. 4). The result is improved accuracy and, especially, higher reliability. The fewer control points are used the more they must be reliable.

Very often blocks are not strictly rectangular but have more irregular shapes. In such cases it is advisable to use additional cross-strips and/or additional ground control points. Examples are the blocks 1, 6, 7, 13, 16, 17, 18, 19 (see fig. 5).

An extreme case is the large block 7 with 1633 photos. There are 39 regular strips, to which essentially 3 cross-strips are added. They are partly broken up, giving altogether 6 cross-strips. In this block 34 control points were used, rather more than would have been neces-

sary. Nevertheless, this block is a prime example for the saving of ground control points by using GPS. This block would have required about 55 horizontal control points and 450 vertical control points, in order to obtain the same accuracy with conventional aerial triangulation.

Tables 1 and 2 also contain blocks with long extension, e.g. blocks 15, 16, 19, 23. The latter consists of 1 strip only. In those cases the conventional ground control pattern is about maintained resp. replaced by cross-strips. But the bridging distances can be considerably larger than in conventional cases.

Another interesting case is block 13. It concerns a large scale block flown with a 60% side overlap and 3 cross-strips, i.e. an extremely stable configuration. The purpose was xyz coordinate determination of signalized points with about 1-2 cm accuracy. The combined block adjustment, based on 4 ground control points only, gave  $\sigma_0 = 4.8 \mu\text{m}$  resp.  $\bar{\sigma}_0 = 2.4 \text{ cm}$ . The empirical r.m.s. value, assessed from 10 horizontal and 86 vertical check points was 1.3 cm ( $= 0.5 \bar{\sigma}_0$ ) in x, 0.8 cm ( $= 0.3 \bar{\sigma}_0$ ) in y and 4.3 cm ( $= 1.8 \bar{\sigma}_0$ ) in z. The respective theoretical values are 1.7 cm, 1.7 cm and 3.4 cm. This result shows that GPS application can be interesting and effective also with large scale blocks, where the expected accuracy is higher than the precision of GPS camera stations. Such cases warrant further investigation, however, as similar results might be obtained without GPS as well.

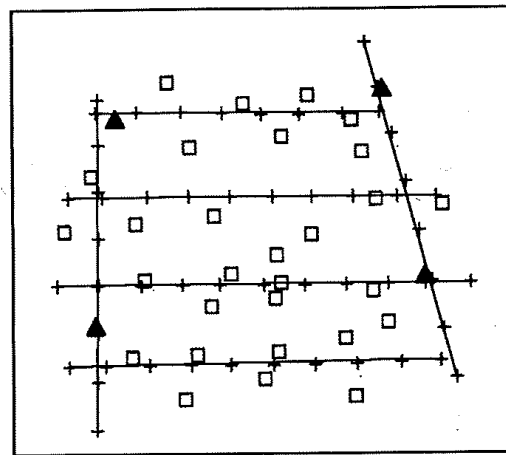


Fig. 3: Example block 4, 2 cross strips, 4 control points

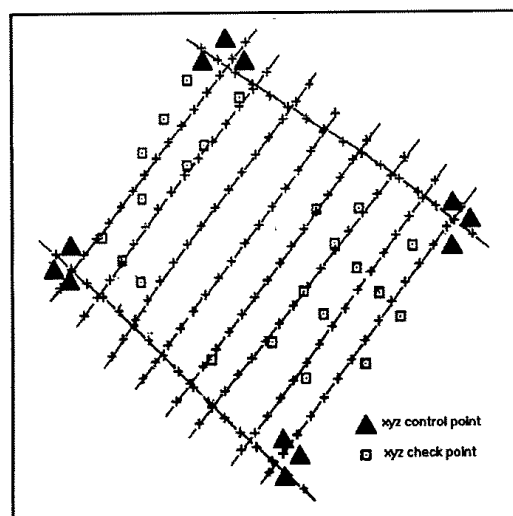


Fig. 4: Example block 11, 2 cross-strips, 4x3 control points

#### 4.3. Stationary Receiver at 400 km Distance

It is one of the main practical advantages of the method of modelling GPS drift errors in the combined block adjustment that the distance between the stationary receiver and the mission area can be very large. The stationary receiver may be located at the home base or at the home office, at a distance of 500 km or more. That situation has been checked with block 11. First, the standard case was calculated with the stationary receiver at the mission area. The photo scale was 1:6100,  $\sigma_0$  had the magnitude of  $6.8 \mu\text{m}$  resp.  $\bar{\sigma}_0 = 4.0 \text{ cm}$ . The absolute accuracy was assessed from 18 xyz check points to 8.2 cm ( $= 2.0 \bar{\sigma}_0$ ) in x,y and 8.7 cm ( $= 2.3 \bar{\sigma}_0$ ) in z. The adjustment was repeated with data from a 2nd GPS receiver which had been placed at a distance of about 400 km. In this case,  $\sigma_0$  was  $6.9 \mu\text{m}$ , resp.  $\bar{\sigma}_0 = 4.1 \text{ cm}$ . The absolute accuracy of the adjusted block, assessed from the same 18 check points, was 8.2 cm ( $= 2.0 \bar{\sigma}_0$ ) in x,y and 6.1 cm ( $= 1.5 \bar{\sigma}_0$ ) in z. The average coordinate differences at all block points between the two block adjustments were 0.3 cm in x,y and 1.7 cm in z. The closer inspection of the data showed, as expected, that the GPS drift

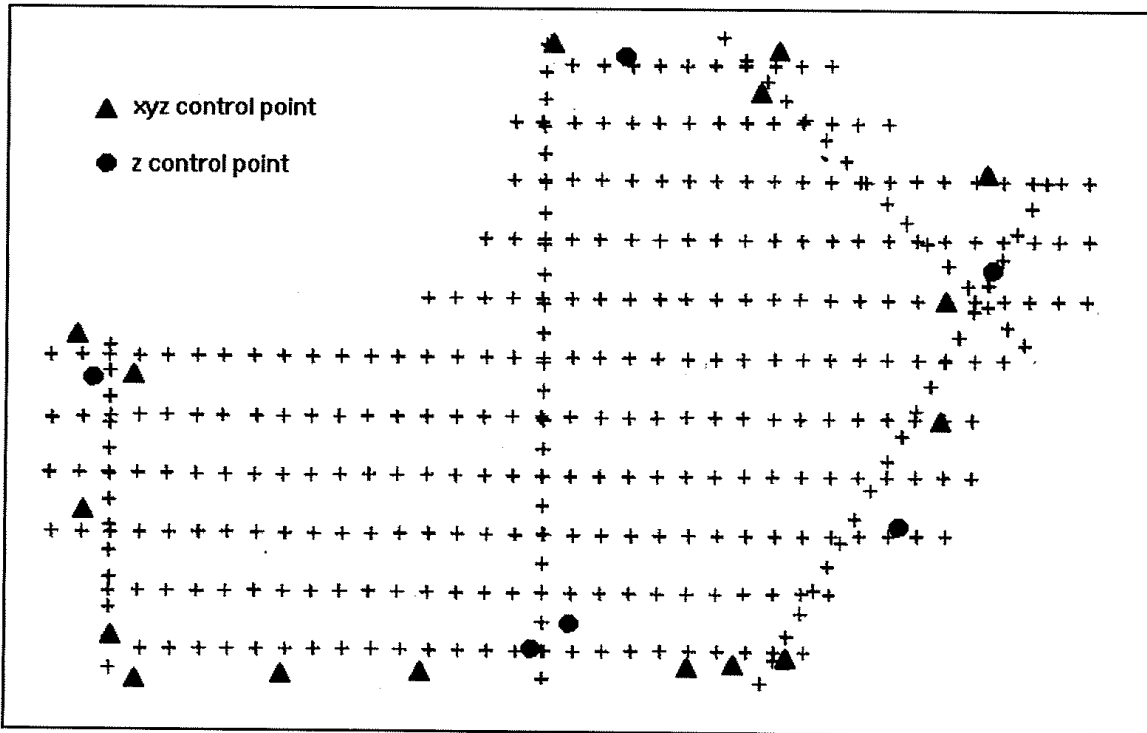


Fig. 5: Example block 1, additional cross-strip and control points

parameters were quite different in both cases, the constant terms differing up to 9 m and 5 m in x and y, resp. up to 27 m in z. However, the drifts were sufficiently linear to be completely compensated by the linear correction terms of the adjustment. This is certainly due to the fact that the corrections are applied per strip and that strips are short, extending in this case to only 2 min. of flying time.

## 5. Economy

The empirical examples have confirmed that GPS supported aerial triangulation works very well, with high accuracy performance, in spite of making use of few control points only. We have, however, also to look into the economy of the system. It is clear that the application of GPS to aerial triangulation is only feasible, if there is an economic advantage, as the same results could be otherwise obtained by conventional aerial triangulation. There are no detailed investigations known, but the general cost situation can be assessed quite comprehensively. 3 cases have been realistically looked into, with the following result: The photogrammetric part of aerial triangulation with GPS is about 25% more expensive than conventionally. That includes: Flying of additional cross-strips, film, GPS equipment, monitoring a GPS ground station, GPS software and GPS trajectory computation, aerial triangulation, point transfer and image coordinate observations (cross-strips included), extended combined block adjustment.

On the other hand GPS blocks operate with few ground control points only. The saving in ground control is in the order of 90% or more, compared with conventional aerial triangulation of the same accuracy. Both positions together result in a net cost saving of about 40% altogether, compared with conventional aerial triangulation, including ground control, flying and aerial triangulation execution. The conclusion is evident that there is certainly sufficient



economic advantage to apply GPS aerial triangulation in practically all cases. The economic advantage decreases only, if ground control points are already given or are inexpensive and when still a considerable number of ground control points may be necessary, as is the case in cadastral photogrammetry, where 1-2 cm accuracy is wanted.

## 6. Conclusion

In this paper the attention has been drawn to the status of practical application of GPS supported aerial triangulation. The examples of tables 1 and 2 show that successful practical applications cover already a wide range of photo scales, of block configurations and of different applications. They also concern different GPS receivers and different photogrammetric companies. The systems are evidently fully operational, in terms of procedure, hardware (GPS) and software. The successful application depends to a great extent on the robust method of allowing GPS drift and datum errors correcting them in combined block adjustment. That method makes the operational conditions of using GPS particularly easy, also by allowing great distances between mission area and stationary GPS receiver. The examples have confirmed the high accuracy performance of GPS blocks. The results can generally be classified as high accuracy aerial triangulation, being close to the photogrammetric measuring precision ( $\bar{\sigma}_0$ ). Also, the empirical accuracy results are close enough to the theoretical expectation to be relied upon in planning GPS aerial triangulation projects.

The economic advantage of GPS supported aerial triangulation is substantial. It is accomplished by the great reduction of ground control points in comparison with conventional aerial triangulation.

In conclusion it can be stated that GPS supported aerial triangulation has quickly outgrown the pilot stage and is considered fully operational in terms of regular practical application. Both the accuracy and the economy performance are such convincing that application is generally recommended for practically all air survey projects.

Very recently improved hardware, e.g. new dual frequency receivers have become available and fast ambiguity solutions promise to make kinematic GPS trajectories more continuous. The method of GPS drift and datum parameters, as described and applied here, will certainly draw benefits of those developments as larger stretches and possibly complete trajectories will have exact ambiguity solutions. Thus the error and datum modelling of GPS data will be considerably simplified and improved and the application of the present software and procedures will become easier. It remains to be investigated, however, how safe, reliable and robust these new generation tools do operate.

## 7. References

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