SOLUTION OF THE INTERIOR ORIENTATION WITHOUT CONTROL POINTS

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Abstract

A new method for solving an unknown interior orientation of a camera using multiple images is presented. This method does not require any control points as a reference but needs at least three images taken with the same camera from distinct standpoints. The solution is based on the connection between the singular correlation, epipolar geometry, and the interior orientation. The interior orientation is solved up to five linear parameters, which are the principal point coordinates, camera constant, non-orthogonality of the image coordinate system, and the scale between the image y- and x-axis.

Key words: Camera calibration, Epipolar geometry, Close-range

1. INTRODUCTION

When non-metric cameras are used for photogrammetric purposes, the unknown interior orientation is usually solved using known object references, thus the geometric constrains are in the object space. This paper presents a method to solve the interior orientation up to five parameters using only the geometric constrains between the images, thus no object reference is needed. The objective is not to perform camera calibration but merely to offer a way to solve the interior orientation on-line along with other computations.

The assumptions concerning the study area are:
- There are at least three images taken with the same camera using separate camera stations. If the three projection centers are collinear, affinity can not be solved but a conventional three-parametric solution is obtained.
- The image coordinates do not define a planar object but are well distributed in the model space in order to get the singular correlations between images computed.

Only linearly deformed images are considered in this method. In practice, linear deformation also exists. For example, video images usually are significantly affine. Usually also interior orientations of image prints are linearly deformed. Possible nonlinear errors in the image coordinates are not modelled.

2. SINGULAR CORRELATION

Singular correlation is a special form of projective transformation between two images. In two-dimensional image space, the singular correlation transforms an image point from the first image to an epipolar line on the second image.

Singular correlation is presented as a single bilinear and homogeneous equation between the image coordinates of two images

$$[\,x'\; y'\; 1\,]\,\mathbf{M}\begin{bmatrix} x'' \\ y'' \\ 1 \end{bmatrix} = 0, \qquad \mathbf{M} = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{bmatrix}, \tag{1}$$

or

$$m_1x'x''+m_2x'y''+m_3x'+m_4y'x''+m_5y'y''+m_6y'+m_7x''+m_8y''+m_9 = 0. \tag{2}$$

The singular correlation matrix **M** contains all essential information about the relative orientation, and something about the interior orientation.

Matrix **M** is singular, thus det(**M**)=0, and it contains only seven independent parameters /Jordan, Eggert, Kneissl/. Thus, using at least seven homolog points, well distributed in space, the entries of **M** are solvable. The solution is found either using a simple linear solution, or a rigorous iterative solution /Haggrén, Niini/. **M** is determined up to an arbitrary scalar factor.

3. EPIPOLES

Epipoles are special image points, particularly existing in a two image system. The projection center of the first image is projected down as the epipole point on the second image, and vice versa. Singular correlation offers a way to find the epipoles, one for each image. Homogeneous epipole coordinates satisfy conditions

$$\mathbf{M}^T\mathbf{e}_1 = \mathbf{0}, \text{ and } \mathbf{M}\,\mathbf{e}_2 = \mathbf{0}. \tag{3}$$

The right side of these equations is a vector of zeroes. The epipoles $\mathbf{e}_1 = [x_1\; y_1\; z_1]^T$ and $\mathbf{e}_2 = [x_2\; y_2\; z_2]^T$ can be computed from these two linear homogeneous equations by fixing one of the three coordinates, and solving the other two. Usually z_1 (resp. z_2) is set to unity but in certain cases it may be zero and some other coordinate should be fixed.

The epipolar points can also be computed from known relative and interior orientation parameters (base $\mathbf{b}=[\,b_x\; b_y\; b_z\,]^T$, rotation **R**, interior orientation **C**):

$$\begin{aligned} \varepsilon_1\mathbf{e}_1 &= \mathbf{C}^{-1}\,\mathbf{b}, \\ \varepsilon_2\mathbf{e}_2 &= \mathbf{R}\,\mathbf{C}^{-1}\,\mathbf{b}. \end{aligned} \tag{4}$$

There is a scale difference between the epipoles obtained from (3) and (4). Therefore the scalar factors ε_1, ε_2 are introduced into above expressions.

Expressions (4) can also be put in matrix form using 3×3 matrix presentation for the base and epipoles:

$$\begin{aligned} \varepsilon_1 \mathbf{E}_1 &= \mathbf{C}^T\, \mathbf{B}\, \mathbf{C}, \\ \varepsilon_2 \mathbf{E}_2 &= \mathbf{C}^T\, \mathbf{R}^T\, \mathbf{B}\, \mathbf{R}\, \mathbf{C}. \end{aligned} \qquad (5)$$

Above, **R** is the conventional orthogonal 3×3 rotation matrix between images. **B** is the skew-symmetric matrix containing the base elements:

$$\mathbf{B} = \begin{bmatrix} 0 & b_z & -b_y \\ -b_z & 0 & b_x \\ b_y & -b_x & 0 \end{bmatrix}.$$

Affine image coordinates are corrected using five-parametric equations

$$\begin{aligned} \mathbf{x} &= \mathbf{x}' + \alpha\, \mathbf{y}' - \mathbf{x}_p, \\ \mathbf{y} &= \beta\, \mathbf{y}' - \mathbf{y}_p, \\ \mathbf{z} &= -\mathbf{c}_p, \end{aligned}$$

which also define the nonsingular interior orientation matrix **C**:

$$\mathbf{C} = \begin{bmatrix} 1 & \alpha & -x_p \\ 0 & \beta & -y_p \\ 0 & 0 & -c_p \end{bmatrix}.$$

α is the non-orthogonality between the image coordinate axes. β is the scale factor of the y-axis with respect to the x-axis. Scale of the x-axis is equal to unity. $\mathbf{x}_p$, $\mathbf{y}_p$ are the principal point coordinates, and $\mathbf{c}_p$ is the camera constant. The upper triangular form of **C** is chosen intentionally.

$\mathbf{E}_1$ is the skew-symmetric epipolar matrix, elements of which are the epipolar coordinates on the first image:

$$\mathbf{E}_1 = \begin{bmatrix} 0 & z_1 & -y_1 \\ -z_1 & 0 & x_1 \\ y_1 & -x_1 & 0 \end{bmatrix}.$$

$\mathbf{E}_2$ is similar to $\mathbf{E}_1$ but contains the epipole of the second image.

4. DEVELOPMENT OF THE METHOD

Singular correlation matrix can be presented as a matrix product

$$\mathbf{M} = \mathbf{C}^T \mathbf{B} \mathbf{R} \mathbf{C}, \tag{6}$$

using the matrices introduced in previous section. Using equations (5), the base matrix **B** can be expressed in two ways

$$\mathbf{B} = \varepsilon_1 \mathbf{C}^{-T} \mathbf{E}_1 \mathbf{C}^{-1},$$

or

$$\mathbf{B} = \varepsilon_2 \mathbf{R}\, \mathbf{C}^{-T} \mathbf{E}_2 \mathbf{C}^{-1} \mathbf{R}^T. \tag{7}$$

Substituting both of them in equation (6), two other forms of **M** are obtained

$$\mathbf{M} = \varepsilon_1 \mathbf{E}_1\, \mathbf{C}^{-1}\, \mathbf{R}\, \mathbf{C}, \tag{8}$$

and

$$\mathbf{M} = \varepsilon_2 \mathbf{C}^T\, \mathbf{R}\, \mathbf{C}^{-T}\, \mathbf{E}_2. \tag{9}$$

By multiplying the equation (8) from right with $\mathbf{C}^{-1}\mathbf{R}^T$, and multiplying equation (9) from left with $\mathbf{C}^{-T}$, modified equations are obtained

$$\mathbf{M}\, \mathbf{C}^{-1} \mathbf{R}^T = \varepsilon_1 \mathbf{E}_1\, \mathbf{C}^{-1}, \tag{10}$$

and

$$\mathbf{C}^{-T}\, \mathbf{M} = \varepsilon_2\, \mathbf{R}\, \mathbf{C}^{-T}\, \mathbf{E}_2. \tag{11}$$

The rotation matrix **R** is eliminated by multiplying the above equations partially with each other. Thus, a single matrix equation follows

$$\varepsilon_2 \mathbf{M}\, \mathbf{C}^{-1} \mathbf{C}^{-T}\, \mathbf{E}_2 = \varepsilon_1 \mathbf{E}_1\, \mathbf{C}^{-1} \mathbf{C}^{-T}\, \mathbf{M}. \tag{12}$$

Note that the arbitrary scale of **M** has no influence on this equation, since **M** appears in both sides of the equation. Denoting $\mathbf{K} = \mathbf{C}^{-1}\mathbf{C}^{-T}$, and $\lambda = \varepsilon_2/\varepsilon_1$, the above equation becomes

$$\lambda\, \mathbf{M}\, \mathbf{K}\, \mathbf{E}_2 = \mathbf{E}_1\, \mathbf{K}\, \mathbf{M}. \tag{13}$$

The objective is now to solve the matrix **K**. Therefore, the structure of the matrix **K** is taken under consideration. Matrix $\mathbf{C}^{-1}$ has the form:

$$\mathbf{C}^{-1} = \begin{bmatrix} 1 & -\dfrac{\alpha}{\beta} & \dfrac{\alpha y_p - \beta x_p}{\beta c_p} \\ 0 & \dfrac{1}{\beta} & -\dfrac{y_p}{\beta c_p} \\ 0 & 0 & -\dfrac{1}{c_p} \end{bmatrix}$$

but the complete form of $\mathbf{K} = \mathbf{C}^{-1}\mathbf{C}^{-T}$ is not displayed here. Instead, the elements of matrix **K** are presented symbolically as

$$\mathbf{K} = \mathbf{C}^{-1}\mathbf{C}^{-T} = \frac{1}{c_p^2}\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \\ \mathbf{b} & \mathbf{d} & \mathbf{e} \\ \mathbf{c} & \mathbf{e} & 1 \end{bmatrix} = \frac{1}{c_p^2}\mathbf{A}.$$

It can be seen from $\mathbf{C}^{-1}$ that the lower right element of **K** is equal to $1/c_p^2$, and therefore it can be taken outside the matrix **A**. Because **K** exists in both sides of the equation (13), the term $1/c_p^2$ is eliminated. Matrix **A** contains five different elements, which are second order functions of the five original interior orientation elements. Elements **a**, **b**, and **d** cannot have whatever arbitrary values but have to satisfy

$$\mathbf{a} > \mathbf{c}^2, \quad \mathbf{b}^2 \geq \mathbf{c}^2\mathbf{e}^2, \quad \text{and} \quad \mathbf{d} > \mathbf{e}^2. \tag{14}$$

The final matrix equation is

$$\lambda\, \mathbf{M}\, \mathbf{A}\, \mathbf{E}_2 = \mathbf{E}_1\, \mathbf{A}\, \mathbf{M}. \tag{15}$$

Denoting $\mathbf{U} = \lambda\, \mathbf{M}\, \mathbf{A}\, \mathbf{E}_2$ and $\mathbf{V} = \mathbf{E}_1\, \mathbf{A}\, \mathbf{M}$, and equating the ratios of elements $\mathbf{u}_{ij}/\mathbf{u}_{33} = \mathbf{v}_{ij}/\mathbf{v}_{33}$ (i, j=1...3), the scalar factor λ is eliminated. There are eight ratios in (15), which give eight possible condition equations for the solution of the interior orientation but only two of them are independent /Maybank, Faugeras/. Thus, at least three equations of kind (15) are needed to solve the five entries of **A**. The equations are naturally formed from at least three different singular correlations, and three images produce just three singular correlations.

The equations between matrix elements in (15) are nonlinear, and they must be linearized but these linearized equations are not presented here, because of their long expressions. The solution of the elements of **A** is iterative, and requires approximate values to be known in advance. Matrices **M**, $\mathbf{E}_1$, and $\mathbf{E}_2$ have to be computed in advance and are kept constants. Computer demonstrations show that the solution converges fast (3-5 iterations) from reasonable approximate values which differ at most ±20 % from the correct values. The approximate values for **a**, **b**, **c**, **d**, and **e** can be obtained from

$$\begin{aligned} \mathbf{a} &= x_p^2 + c_p^2, \\ \mathbf{b} &= x_p y_p, \\ \mathbf{c} &= x_p, \\ \mathbf{d} &= y_p^2 + c_p^2, \\ \mathbf{e} &= y_p, \end{aligned} \tag{16}$$

where x_p, y_p, and c_p are approximate principal point and camera constant (usually the center of the image and the nominal focal length). Equations (16) present exact relationships between the elements of **A** and the original interior orientation parameters if the affinity parameters really are $\alpha = \mathbf{0}$, $\beta = \mathbf{1}$. Then the equations (16) also impose two orthogonality conditions between the entries of **A**:

$$\mathbf{a} - \mathbf{d} = \mathbf{c}^2 - \mathbf{e}^2, \quad \text{and} \quad \mathbf{b} = \mathbf{c}\,\mathbf{e}. \tag{17}$$

Solving system (15) with these two conditions the solution of the interior orientation matrix is three-parametric (orthogonal):

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & -x_p \\ 0 & 1 & -y_p \\ 0 & 0 & -c_p \end{bmatrix}.$$

Only two singular correlations are needed to solve the orthogonal interior orientation but at least three images are still required to obtain the two distinct singular correlations to be used in computations. Orthogonality conditions also make the non-collinearity requirement of the three projection centers insignificant.

4.1 SOLUTION OF C

After the entries of the matrix **A** have been computed, the original interior orientation matrix **C** is obtained from **A** using matrix computations:

$$\mathbf{K}' = \mathbf{A}^{-1},$$

$$\mathbf{Q} = \frac{1}{k'_{11}}\mathbf{K}',$$

$$\mathbf{C} = \mathbf{Chol}(\mathbf{Q}).$$

Scalar $\mathbf{k}'_{11}$ is the upper left element of $\mathbf{K}'$. **Chol(Q)** means Cholesky factorization, which can be used, since **Q** (= $\mathbf{C}^T\mathbf{C}$) should always be a positive definite matrix if conditions (14) are satisfied. The resulting matrix **C** will then be an upper triangular matrix. Explicit solutions for the interior orientation elements are

$$\alpha = \frac{c\,e - b}{d - e^2},$$

$$\beta = \frac{1}{d - e^2}\sqrt{(a - c^2)(d - e^2) - (c\,e - b)^2},$$

$$x_p = \frac{c\,d - b\,e}{d - e^2},$$

$$y_p = \frac{e}{d - e^2} \sqrt{(a - c^2)(d - e^2) - (ce - b)^2} = e\,\beta ,$$

$$c_p = \sqrt{a - c^2 - \frac{(ce - b)^2}{d - e^2}} \; .$$

After the interior orientation has been computed, also the relative orientations of the images used in the computation can be solved using conventional iterative methods or closed form solutions. There are numerous algorithms known in photogrammetry to do this task and therefore it is not examined here.

5. CONCLUSIONS

A new way to compute an unknown interior orientation of a camera is presented. The method is developed assuming the images free from nonlinear errors, and without any use of known object references. The interior orientation is possible to be solved up to five linear parameters starting from reasonable approximate values. The solution is found from the connection between singular correlation, interior orientation, and the epipole points of the images. The connection is presented in the form of matrix equations.

One singular correlation gives only two independent conditions for the interior orientation, thus at least three singular correlations or three images have to be used. The images have to be taken from different positions so that the projection centers are not collinear, in order to get the affinity computed. Computer simulations show that the method already works satisfactorily. Further studies are being made conserning the accuracy and stability of the method in various cases.

The single camera case is simple and also practical in many real world applications and the method can be modified to include also cases where the images have partially or completely different interior orientation parameters.

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