RELATIVE ORIENTATION OF MULTIPLE IMAGES USING PROJECTIVE SINGULAR CORRELATION

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KEY WORDS: Close-range, Photogrammetry, Relative orientation, Camera calibration, 3-D reconstruction, CAD/CAM

Abstract

A relative orientation method of multiple images with unknown interior orientations is presented. The method is based on the projective singular correlation between the images and on the relations between the singular correlation, relative orientation, epipolar geometry, and interior orientation. The method does not require any control points as a reference, but at least three images taken with the same camera from distinct standpoints are required. The computations are linear in principle, and only the solution of the interior orientation is iterative and requires approximate values which also can be obtained linearly from a set of three images taken from the same position in space. The relative orientation method has four steps. First, all singular correlations between the images are computed and the epipoles are extracted from the correlations. Second, the interior orientations of the images are computed. Third, the relative orientations of the images are computed. Finally, the model coordinates of the object are obtained using the relative orientation parameters. The method can be applied for 3-D object reconstruction in a CAD/CAM environment.

1. INTRODUCTION

In conventional photogrammetry, it is usually hard to obtain approximate values for the collinearity-based solution methods except in standard aerial photography. The problem is even more obvious in close-range applications where camera positions may be arbitrary, and cameras with unknown interior orientations are used and, especially, when no control points are available. Therefore, the need to obtain the block and the object geometry directly from the image data is evident.

The paper was presented to the ISPRS Commission III Symposium, Munich 1994, and originally published in the International Archives of Photogrammetry and Remote Sensing Vol. 30, Part 3/2, pp. 615-621.

In this paper, a method for relative orientation of multiple images is presented which solves the problem directly without any prior knowledge of the object geometry or the orientation of the images. The method is based on the projective singular correlations between the images, and needs at least three images taken with the same camera using separate camera stations. More images with different interior orientation parameters can also be used to strengthen the block geometry.

The computations are linear in principle, and only the solution of the unknown interior orientation is iterative and requires approximate values to be known in advance. Even these values can be obtained linearly if additional three images have been taken from the same spatial position using different rotations. Usually, however, the nominal focal length, and the centre of the image give suitable approximate values for the iterative solution of the interior orientation.

The transformation from the object to the images is assumed to be projectively linear, thus nonlinear distortions are not considered or they have to be corrected in advance. Then there are at most five linear interior orientation parameters in each unknown camera. The relative orientation parameters are the rotation matrices, and the base components of each image with respect to the first, reference image which is kept fixed to determine the model coordinate system. The resulting model will be in an orthogonal three- dimensional coordinate system but in an arbitrary scale.

Because the computation of the singular correlations requires significant structure or depth in the object space, the method is not directly applicable in aerial photography but is extremely suitable for close-range applications with arbitrary camera orientations.

2. SINGULAR CORRELATION

The singular correlation between two images is the core of the presented method, because it contains all essential information about the relative orientation and something about the interior orientation of the two correlated images. Therefore, multiple singular correlations are needed to solve for all unknown parameters in the block. Singular correlation is also called the epipolar transformation /Maybank et al., 1992/, because it transforms an image point from the first image to an epipolar line in the second image.

The singular correlation equation between homolog image points of two images is

$$[\,x'\;y'\;1\,]\,M\begin{bmatrix}x''\\y''\\1\end{bmatrix}=0,\quad M=\begin{bmatrix}m_1 & m_2 & m_3\\m_4 & m_5 & m_6\\m_7 & m_8 & m_9\end{bmatrix}. \tag{1}$$

This is in short $\mathbf{x}_1^T\mathbf{M}\mathbf{x}_2 = \mathbf{0}$ or also $\mathbf{x}_2^T\mathbf{M}^T\mathbf{x}_1 = \mathbf{0}$. The equation is linear in terms of the nine correlation coefficients. The 3×3 correlation matrix **M** is singular, thus det(**M**)=0, and it contains only seven independent parameters /Jordan et al., 1972/. Using at least seven homolog points, well distributed in space, the elements of **M** can be solved. It is, however, possible to determine **M** only up to an arbitrary scale factor since the equation (1) is homogeneous.

It has been shown in /Niini, 1994/ that the singular correlation condition can be derived from the coplanarity condition of the conventional relative orientation. Then the correlation matrix is the matrix product of the relative orientation parameters as follows

$$\tau_{12}\mathbf{M}_{12} = \mathbf{C}_1^T \mathbf{B}_{12} \mathbf{R}_2 \mathbf{C}_2, \tag{2}$$

where τ_{12} is the arbitrary scale of $\mathbf{M}_{12}$. $\mathbf{C}_1$ and $\mathbf{C}_2$ are the nonsingular interior orientation matrices of the two images including the affinity of the image. They have the form

$$C = \begin{bmatrix} 1 & \alpha & -x_p \\ 0 & \beta & -y_p \\ 0 & 0 & -c_p \end{bmatrix}, \tag{3}$$

where $\mathbf{x}_p$, $\mathbf{y}_p$ is the principal point, $\mathbf{c}_p$ is the camera constant, or the focal length, and the affinity parameters α, β are the skewness of the image coordinate system, and the scale ratio of the image coordinate axes, respectively. Naturally, these values can be different in each camera. Matrix $\mathbf{B}_{12}$ is the singular skew-symmetric base matrix

$$B_{12} = \begin{bmatrix} 0 & b_z & -b_y \\ -b_z & 0 & b_x \\ b_y & -b_x & 0 \end{bmatrix}, \tag{4}$$

containing the three base components. The standard orthogonal 3×3 rotation matrix between the two images is denoted with $\mathbf{R}_2$, and the first image is assumed to be fixed in space with $\mathbf{R}_1=\mathbf{I}$ (identity matrix) and zero projection centre coordinates.

It is well known that the problem of relative orientation of two images contains only five parameters assuming known interior orientations, thus a known singular correlation with seven parameters can contain only two conditions for unknown interior orientations $\mathbf{C}_1$ and $\mathbf{C}_2$.

3. EPIPOLES

The epipoles have an important function in the presented method since they contain certain information both about the interior and the relative orientation parameters.

In relative orientation, the projection centre of the first image is projected as the epipole point on the second image, and vice versa. The epipoles are the intersection points of the epipolar lines which were produced under the singular correlation. Both epipoles can be solved from a known singular correlation matrix. Epipole coordinates satisfy conditions

$$\mathbf{M}_{12}{}^{T}\mathbf{e}_{12} = \mathbf{0} \quad \text{and} \quad \mathbf{M}_{12}\,\mathbf{e}_{21} = \mathbf{0}. \tag{5}$$

The epipoles $\mathbf{e}_{12} = [\mathbf{x}_1\ \mathbf{y}_1\ \mathbf{z}_1]^T$ and $\mathbf{e}_{21} = [\mathbf{x}_2\ \mathbf{y}_2\ \mathbf{z}_2]^T$ can be computed from these two linear homogeneous equations by fixing one of the coordinates, say $\mathbf{z}$, to a preset value.

It has been shown in /Niini, 1994/ that the epipoles are related to the relative and interior orientation parameters by equations

$$\varepsilon_{12}\mathbf{E}_{12} = \mathbf{C}_1{}^T\,\mathbf{B}_{12}\,\mathbf{C}_1 \tag{6}$$

and

$$\varepsilon_{21}\mathbf{E}_{21} = \mathbf{C}_2{}^T\,\mathbf{R}_2{}^T\,\mathbf{B}_{12}\,\mathbf{R}_2\,\mathbf{C}_2, \tag{7}$$

using the previously introduced matrices. $\mathbf{E}_{12}$ is the skew-symmetric epipolar matrix, elements of which are the coordinates of the epipole in the first image:

$$E_{12} = \begin{bmatrix} 0 & z_1 & -y_1 \\ -z_1 & 0 & x_1 \\ y_1 & -x_1 & 0 \end{bmatrix}.$$

$\mathbf{E}_{21}$ is similar to $\mathbf{E}_{12}$ but contains the coordinates of the epipole in the second image. There is a scale difference between the epipoles obtained from (5) and (6),(7). Therefore the scalar factors ε_{12}, ε_{21} are introduced in equations (6) and (7).

The standard relative orientation condition is

$$\lambda_1\mathbf{C}_1\mathbf{x}_1 = \lambda_2\mathbf{R}_2\mathbf{C}_2\mathbf{x}_2 + \mathbf{b}_{12}, \tag{8}$$

where λ_1 and λ_2 are the projection ray scales of the two homolog image points, and $\mathbf{b}_{12}$ is the base vector. Multiplying both sides of (8) from the left with $\varepsilon_{12}\mathbf{E}_{12}\mathbf{C}_1{}^{-1}$ an additional relation between the epipole matrix and the singular correlation matrix is obtained. Actually, the following relation only expresses the same epipolar line in two ways

$$\lambda_1\mathbf{E}_{12}\mathbf{x}_1 = \lambda_2\delta_{12}\mathbf{M}_{12}\mathbf{x}_2. \tag{9}$$

The equality $\tau_{12}\mathbf{M}_{12}=\varepsilon_{12}\mathbf{E}_{12}\mathbf{C}_1{}^{-1}\mathbf{R}_2\mathbf{C}_2$ has been used here.
$\delta_{12}=\tau_{12}/\varepsilon_{12}$ is the scale between the matrices $\mathbf{M}_{12}$ and $\mathbf{E}_{12}$ /Niini, 1994/. This will be used later to determine the rotation between the images. Unfortunately, the value of δ_{12} can be computed only after the interior orientations are known.

4. THE RELATIVE ORIENTATION METHOD

The relative orientation method is divided into four steps: the solution of the singular correlations and the epipoles, the solution of the interior orientations, the solution of the rotations, and the solution of the base vectors along with the model coordinates.

Assume that there are **k** images taken from the same object using different standpoints and perhaps using different cameras. At least three of the images have to be taken with the same camera, or alternatively, four images have to be taken with two different cameras (also two images with each camera). There are at most $(\mathbf{k}^2-\mathbf{k})/2$ possible image pairs or singular correlations to be solved. All possible correlations are not necessarily needed if the number of images is large. Each image should be contained in at least three different singular correlations in order to obtain sufficient number of redundancy for the solution of the interior orientations, and the above mentioned three or four images have to be contained among these correlations.

The singular correlation matrices are not completely independent but the three singular correlations $\mathbf{M}_{12}$, $\mathbf{M}_{13}$, and $\mathbf{M}_{23}$ between three images are always related via $\mathbf{M}_{12}\mathbf{e}_{23}=\mathbf{M}_{13}\mathbf{e}_{32}$. The epipoles $\mathbf{e}_{23}$ and $\mathbf{e}_{32}$ are obtained from the correlation $\mathbf{M}_{23}$, thus the three correlations have three dependencies between them. These conditions can be introduced as additional conditions in the solution of the singular correlations and the epipoles.

Only one singular correlation is sufficient to solve for the relative orientation parameters after the interior orientations have been solved.

4.1 Solution of the singular correlations

The equations of type (1) are formed for all correlated points and for all correlated image pairs. Then the unknown correlations are $\mathbf{M}_{12}$, $\mathbf{M}_{13}$, ..., $\mathbf{M}_{ij}$, ..., $\mathbf{M}_{k-1,k}$, $\mathbf{i=1...k-1}$, $\mathbf{j=i+1...k}$, which have to be solved simultaneously in a block adjustment. Linearization of the equations and the determinant constraints gives a standard least squares system of equations with constraints as follows

$$\begin{aligned} \mathbf{Am} + \mathbf{Bv}_x &= \mathbf{l}, \\ \mathbf{Cm} &= \mathbf{g}. \end{aligned} \qquad (10)$$

Here, the adjustment is made by minimization of the residuals of the original image coordinates. The first system contains the partial derivatives of the unknowns (matrix A) and the observations (matrix B) from equations (1), and the latter system contains the partial derivatives of the unknowns (matrix C) from the determinant conditions. The solution of such a system can be found from /Mikhail, 1976/.

The approximate values needed for the iterative solution are obtained by solving each singular correlation separately from the linear equations (1) using at least eight pairs of homolog points /Haggrén et al., 1990/.

After the singular correlation have been computed, the corresponding epipoles are also known from the equations (5).

4.2 Solution of the interior orientation

The solution of the interior orientation without control points was presented originally in /Niini, 1993/ but the main idea of the method is presented also here for completeness. The equations are derived using images 1 and 2 but the result is valid in general.

Using equations (6) and (7), the base matrix $\mathbf{B}_{12}$ between images 1 and 2 can be solved in two different ways

$$\mathbf{B}_{12} = \varepsilon_{12}\mathbf{C}_1^{-T}\mathbf{E}_{12}\mathbf{C}_1^{-1}$$

or **(11)**

$$\mathbf{B}_{12} = \varepsilon_{21}\mathbf{R}_2\,\mathbf{C}_2^{-T}\mathbf{E}_{21}\mathbf{C}_2^{-1}\mathbf{R}_2^{T}.$$

Substituting them into equation (2) in turn, two presentations of the correlation $\mathbf{M}_{12}$ are obtained

$$\tau_{12}\mathbf{M}_{12} = \varepsilon_{12}\mathbf{E}_{12}\,\mathbf{C}_1^{-1}\,\mathbf{R}_2\,\mathbf{C}_2$$

and **(12)**

$$\tau_{12}\mathbf{M}_{12} = \varepsilon_{21}\mathbf{C}_1^{T}\,\mathbf{R}_2\,\mathbf{C}_2^{-T}\,\mathbf{E}_{21}.$$

These can be modified further to

$$\tau_{12}\mathbf{M}_{12}\,\mathbf{C}_2^{-1}\mathbf{R}_2^{T} = \varepsilon_{12}\mathbf{E}_{12}\,\mathbf{C}_1^{-1}$$

and **(13)**

$$\tau_{12}\mathbf{C}_1^{-T}\,\mathbf{M}_{12} = \varepsilon_{21}\,\mathbf{R}_2\,\mathbf{C}_2^{-T}\,\mathbf{E}_{21}.$$

The rotation matrix $\mathbf{R}_2$ is eliminated by multiplying the above equations pairwise with each other. A single matrix equation follows

$$\varepsilon_{21}\mathbf{M}_{12}\,\mathbf{C}_2^{-1}\mathbf{C}_2^{-T}\,\mathbf{E}_{21} = \varepsilon_{12}\mathbf{E}_{12}\,\mathbf{C}_1^{-1}\mathbf{C}_1^{-T}\,\mathbf{M}_{12}. \quad \mathbf{(14)}$$

Note that the arbitrary scale of $\mathbf{M}_{12}$ has no influence on (14) since $\mathbf{M}_{12}$ appears in both sides of the equation. Denoting $\mathbf{K}_1 = \mathbf{C}_1^{-1}\mathbf{C}_1^{-T}$, $\mathbf{K}_2=\mathbf{C}_2^{-1}\mathbf{C}_2^{-T}$, and $\lambda_{12}=\varepsilon_{21}/\varepsilon_{12}$, the equation becomes

$$\lambda_{12}\,\mathbf{M}_{12}\,\mathbf{K}_2\,\mathbf{E}_2^{1} = \mathbf{E}_{12}\,\mathbf{K}_1\,\mathbf{M}_{12}. \quad \mathbf{(15)}$$

The elements of matrix $\mathbf{K}_1$ can be presented symbolically as

$$K_1 = C_1^{-1}C_1^{-T} = \frac{1}{c_{pi}^2}\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & 1 \end{bmatrix} = \frac{1}{c_{pi}^2}A_1.$$

$\mathbf{K}_2$ has similar structure. Whatever the other interior orientation parameter are, the lower right element of each $\mathbf{K}_i$ (i=1...k) depends only on the camera constant (as $1/c_{pi}^2$). Therefore it can be taken outside the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$, and their ratio can be joined to the scalar λ_{12} like in equation (17). This fixes the lower right element of $\mathbf{A}_i$ always to unity. Then each matrix $\mathbf{A}_i$ contains five different elements which are second order functions of the five original interior orientation elements of the camera.

The elements $\mathbf{a}_i$, $\mathbf{b}_i$, and $\mathbf{d}_i$ cannot have whatever arbitrary values but have to satisfy

$$\mathbf{a}_i > \mathbf{c}_i^2, \quad \mathbf{b}_i^2 \geq \mathbf{c}_i^2\mathbf{e}_i^2, \quad \text{and} \quad \mathbf{d}_i > \mathbf{e}_i^2, \tag{16}$$

in order to ensure that the matrix product $\mathbf{C}_i^T\mathbf{C}_i$ will be positive definite. This requirement comes from the fact that Cholesky factorization will be used to solve the original $\mathbf{C}_i$ from the solved $\mathbf{A}_i$.

The final matrix equation is

$$\boldsymbol{\mu}_{12}\, \mathbf{M}_{12}\, \mathbf{A}_2\, \mathbf{E}_{21} = \mathbf{E}_{12}\, \mathbf{A}_1\, \mathbf{M}_{12}, \tag{17}$$

where $\boldsymbol{\mu}_{12}=\lambda_{12}\mathbf{c}_{p1}^2/\mathbf{c}_{p2}^2$. The form of the equation is valid for any pair of images if the indexes 1 and 2 are replaced with i and j, and the epipoles are indexed accordingly.

Both sides of the equation (17) actually are 3×3 matrices, and the equation can be presented as $\boldsymbol{\mu}_{12}\mathbf{U}=\mathbf{V}$. The best way to continue is perhaps the elimination of $\boldsymbol{\mu}_{12}$ by dividing the left and right sides with one of the matrix elements, say the lower right element, giving eight equations of type $\mathbf{u}_{ij}/\mathbf{u}_{33}=\mathbf{v}_{ij}/\mathbf{v}_{33}$. The equations contain only five (if the images have the same interior orientation) or ten parameters. The equations obtained are not presented here. The eight equations are nonlinear in the unknowns $\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$, $\mathbf{d}_i$, and $\mathbf{e}_i$, **i=1,2**. The equations, however, contain only two true conditions for the unknown interior orientations. Therefore a sufficient number of image pairs are needed to obtain a solution for the interior orientations.

If $\mathbf{A}_1=\mathbf{A}_2=\mathbf{A}_3$, or the interior orientation is the same in three images, then also three matrix equations are obtained among these images which are sufficient for the solution of the five interior orientation parameters. If the projection centres of the three images are collinear, affinity cannot be solved. There has to be some rotation between the images to guarantee a good determinability of the camera constant. Also the bases between the images should be approximately equilaterally positioned around the object in order to get a consistent solution for the common interior orientation /Niini, 1994/.

Multiple images with different interior orientations can be added to the system simply by forming the corresponding equations (17) between the new image and the three original images.

Computer demonstrations show that the iterative solution converges satisfactorily starting from approximate values which differ at most ±20 % from the correct values. The initial values for $\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$, $\mathbf{d}_i$, and $\mathbf{e}_i$ of $\mathbf{A}_i$ can be obtained from the approximately known interior orientation parameters $\mathbf{x}_{pi}$, $\mathbf{y}_{pi}$, and $\mathbf{c}_{pi}$ of the image by

$$\begin{aligned}
\mathbf{a}_i &= \mathbf{x}_{pi}^2 + \mathbf{c}_{pi}^2, \\
\mathbf{b}_i &= \mathbf{x}_{pi}\mathbf{y}_{pi}, \\
\mathbf{c}_i &= \mathbf{x}_{pi}, \\
\mathbf{d}_i &= \mathbf{y}_{pi}^2 + \mathbf{c}_{pi}^2, \\
\mathbf{e}_i &= \mathbf{y}_{pi}.
\end{aligned} \qquad \textbf{(18)}$$

4.2.1 Special cases. Equations (18) are exact in the case where affinity does not exist, i.e. $\alpha_i = \mathbf{0}$, $\beta_i = \mathbf{1}$. Then the equations also impose two orthogonality conditions between the entries of $\mathbf{A}_i$:

$$\mathbf{a}_i - \mathbf{d}_i = \mathbf{c}_i^2 - \mathbf{e}_i^2 \quad \text{and} \quad \mathbf{b}_i = \mathbf{c}_i\,\mathbf{e}_i. \qquad \textbf{(19)}$$

Using these conditions as constraints to the elements of $\mathbf{A}_i$ with equations (17) the solution of the corresponding interior orientation matrix will have a three-parametric form:

$$C_i = \begin{bmatrix} 1 & 0 & -x_p \\ 0 & 1 & -y_p \\ 0 & 0 & -c_p \end{bmatrix}.$$

Further, it is also possible to treat the interior orientation parameters only partially different between images, even though they were originally taken with the same camera. For example, the use of a comparator for measurements may cause different principal point coordinates but preserve the same camera constant in the images. In a general case the additional condition between the elements of $\mathbf{A}_i$ and $\mathbf{A}_j$ of images **i** and **j** is then

$$\begin{aligned}
&(\mathbf{a}_i - \mathbf{c}_i^2)(\mathbf{d}_i - \mathbf{e}_i^2) - (\mathbf{c}_i\mathbf{e}_i - \mathbf{b}_i)^2 \\
&= (\mathbf{a}_j - \mathbf{c}_j^2)(\mathbf{d}_j - \mathbf{e}_j^2) - (\mathbf{c}_j\mathbf{e}_j - \mathbf{b}_j)^2.
\end{aligned} \qquad \textbf{(20)}$$

Ignoring affinity parameters, it reduces to

$$\mathbf{a}_i - \mathbf{c}_i^2 = \mathbf{a}_j - \mathbf{c}_j^2. \qquad \textbf{(21)}$$

These relations may hold between several images. Note that the condition (21) has to be used together with the conditions (19) to work properly.

4.2.2 Solution of $\mathbf{C}_i$. After the entries of the matrix $\mathbf{A}_i$ have been computed, the original interior orientation matrix $\mathbf{C}_i$ is obtained from $\mathbf{A}_i$ using matrix computations:

$$K'_i = A_i^{-1},$$

$$Q_i = \frac{1}{k'_{i11}} K'_i, \qquad (22)$$

$$C_i = \mathrm{Chol}(Q_i).$$

Scalar $\mathbf{k'}_{i11}$ is the upper left element of $\mathbf{K}_i'$. $\mathbf{Chol(Q_i)}$ means Cholesky factorization which can be used since $\mathbf{Q}_i$ (= $\mathbf{C}_i^T\mathbf{C}_i$) should always be a positive definite matrix if conditions (16) are satisfied. The resulting matrix $\mathbf{C}_i$ will then be an upper triangular matrix as it should be by definition. Explicit solutions for the interior orientation elements directly from the matrix $\mathbf{A}_i$ can be found in /Niini, 1993/.

4.2.3 Linear solution of the interior orientation. If there are three images with the same interior orientation taken from the same spatial position but with different rotations, it is possible to solve the interior orientation using pure linear equations /Hartley, 1994/. The principle is applicable also to solve for the approximate interior orientation elements of a camera if the base between images is small compared to the distance to the object.

Assuming $\mathbf{C=C_1=C_2}$, and a zero base, the projection rays of homolog points of two images are related as

$$\mathbf{Cx_1 = R_2Cx_2}. \qquad (23)$$

The commonly used projection ray scales can be ignored here because there is no parallax between the images. Relation (23) reduces, instead of the singular correlation, to the 2-D projective transformation

$$\mathbf{x_1 = P\ x_2} \qquad (24)$$

with the transformation matrix $\mathbf{P = C^{-1}R_2C}$. Using at least four homolog points $\mathbf{P}$ can be solved up to an unknown scale. It is easily checked that the determinant of $\mathbf{P}$ is exactly one if the definition of $\mathbf{C}$ is like (3) and $\mathbf{R}_2$ is an orthogonal rotation matrix. This makes it possible to scale the matrix $\mathbf{P}$ accordingly before further computations. Again, elimination of $\mathbf{R}_2$ from $\mathbf{P = C^{-1}R_2C}$ gives

$$\mathbf{PC^{-1}C^{-T}P^T = C^{-1}C^{-T}}, \qquad (25)$$

or

$$\mathbf{P\ A\ P^T = A} \qquad (26)$$

which is linear in the elements of **a**, **b**, **c**, **d**, and **e**. The initial values of them can now be solved using at least three equations of type (26) using images which have been taken approximately from the same position in space compared to the object distance. Again, the images should be clearly rotated with respect to each others in order to get the interior parameters determined. If needed at this stage, the solution of the actual interior orientation parameters is obtained using the Cholesky factorization algorithm (22).

Note that the equation (25) could have been formed to solve for $\mathbf{A'}=\mathbf{C}^T\mathbf{C}$ instead which might be more appropriate because the inversion of **A'** would then not be needed using algorithm (22), but the current way allows a direct solution of the approximate values needed in further computations with equations (17).

4.3 Solution of the rotation matrix

After the interior orientations $\mathbf{C}_1$, $\mathbf{C}_2$ have been solved the absolute value of δ_{12} from (9) can be determined from

$$\delta_{12} = |\mathbf{E}_{12}\mathbf{C}_1^{-1}| / |\mathbf{M}_{12}\mathbf{C}_2^{-1}| \qquad (27)$$

using the Frobenius' norm of the matrices /Niini, 1994/. The correct sign of δ_{12} can then be determined using a freely chosen homolog point with image coordinates $\mathbf{x}_1$, and $\mathbf{x}_2$. The sign of δ_{12} is determinable using expression (9) because λ_1 and λ_2 have always the same positive sign if the model is assumed to lie in front of the positive images.

The correct rotation matrix between two images, the reference image and the image under consideration, is found from the relation

$$\tau\mathbf{M}_{12} = \mathbf{C}_1^T\mathbf{B}_{12}\mathbf{R}_2\mathbf{C}_2. \qquad (28)$$

By substituting $\mathbf{B}_{12} = \varepsilon_{12}\mathbf{C}_1^{-1}\mathbf{E}_{12}\mathbf{C}_1^{-1}$ into the above equation, and after some manipulations, the following result is obtained

$$\delta_{12}\,\mathbf{C}_1^{-T}\mathbf{M}_{12}\mathbf{C}_2^{-1} = \mathbf{C}_1^{-T}\mathbf{E}_{12}\mathbf{C}_1^{-1}\mathbf{R}_2, \qquad (29)$$

or, using a shorter notation

$$\mathbf{N} = \mathbf{D}\,\mathbf{R}_2. \qquad (30)$$

Because the ratio $\delta_{12} = \tau_{12}/\varepsilon_{12}$ was computed earlier, both **N** and **D** are known exactly. **N** is a reduced correlation matrix, and **D** is a temporary skew-symmetric base matrix which differs from the true base matrix only by an unknown scalar factor. The corresponding temporary base vector is denoted with **d**, with elements $\mathbf{d}_x$, $\mathbf{d}_y$, and $\mathbf{d}_z$ taken out from **D**. The rotation matrix is computed explicitly from

$$\mathbf{R}_2 = (\mathbf{N}^{*T} + \mathbf{D}^T\mathbf{N})/\mathbf{d}^2. \qquad (31)$$

where $\mathbf{N}^*$ is the adjoint (adjugate) matrix of **N**. Note that all elements of $\mathbf{R}_2$ are solved simultaneously without referring to the individual angular quantities. The scaling term $\mathbf{d}^2$ is simply

$$\mathbf{d}^2 = \mathbf{d}_x^2 + \mathbf{d}_y^2 + \mathbf{d}_z^2 \qquad (32)$$

The rotations of the images j=2...k with respect to the reference image 1 can be computed in the way presented above using, of course, the corresponding singular correlation matrix $\mathbf{M}_{1j}$, interior orientation $\mathbf{C}_j$, and the epipole $\mathbf{E}_{1j}$.

4.4 Determining the base components

The easiest way to obtain the initial base vectors between the reference image and the other images is to apply the first equation of (11) to compute base $\mathbf{B}_{1j}$, and hence $\mathbf{b}_{1j}$, using the solved interior orientation of the first image and the corresponding epipole. The matrix **D** computed in previous chapter is just this base matrix. However, only the ratios of the base components are obtained.

The determination of the correct scales of all other bases except one of them is left in the last stage which is the solution of the model coordinates. One of the bases, say $\mathbf{b}_{12}$, have to be chosen as a reference which determines the overall scale of the entire model. It can also be scaled to a predefined value if needed. The correct direction of this reference base has to be determined at first.

Again, the relation (8) or

$$\lambda_1 \mathbf{C}_1 \mathbf{x}_1 = \lambda_2 \mathbf{R}_2\ \mathbf{C}_2\ \mathbf{x}_2 + \mathbf{b}_{12}$$

is used. The projection ray scales λ_1, λ_2 are always positive since the object is assumed to lie in front of the positive images. Choosing λ_1=1 and computing $\lambda_2 = |\lambda_1 \mathbf{E}_1 \mathbf{x}_1| / |\delta_{12} \mathbf{M}_{12} \mathbf{x}_2|$ with a positive sign, a vector $\mathbf{t} = \mathbf{C}_1\mathbf{x}_1 - \lambda_2 \mathbf{C}_2 \mathbf{x}_2$ is obtained which tells the correct signs of the base components of $\mathbf{b}_{12}$.

4.5 Base scales and model coordinates

The scales of the bases are adjusted finally along with the computation of the model coordinates. The computed interior orientations, and rotations are kept fixed, and $\mathbf{b}_{12}$ is kept constant which then determines the final scale of the entire block. An unknown scale factor β_j is attached to all other base vectors. The model coordinates and unknown scales of the bases can then be solved using linear techniques.

The model coordinates $\mathbf{X'}_i = [\mathbf{X}_i\ \mathbf{Y}_i\ \mathbf{Z}_i]^T$ are determined from all the images where the model point is observed using equations:

$$\begin{aligned}
\lambda_{i1}\ \mathbf{C}_1\mathbf{x}_1\ &= \mathbf{X'}_i, \\
\lambda_{i2}\mathbf{R}_2\ \mathbf{C}_2\mathbf{x}_2 &= \mathbf{X'}_i - \mathbf{b}_{12}, \\
\lambda_{i3}\mathbf{R}_3\ \mathbf{C}_3\mathbf{x}_3 &= \mathbf{X'}_i - \beta_3\mathbf{b}_{13}, \\
&\vdots \\
\lambda_{ij}\mathbf{R}_j\ \mathbf{C}_j\mathbf{x}_j &= \mathbf{X'}_i - \beta_j\mathbf{b}_{1j}.
\end{aligned} \tag{33}$$

λ_{ij} is the scale of the projection ray of the point $\mathbf{X'}_i$ from image j. Elimination of the projection ray scales $\lambda_{i1}, \lambda_{i2}, \lambda_{i3}, \ldots, \lambda_{ik}$ gives the following equations which are linear with respect to the unknown model coordinates $\mathbf{X}_i$, $\mathbf{Y}_i$, $\mathbf{Z}_i$ and the scale unknowns β_j:

$$\begin{aligned} &\mathbf{X_i a_{ij} + Y_i b_{ij} + Z_i c_{ij} - \beta_j (b_{xj} a_{ij} + b_{yj} b_{ij} + b_{zj} c_{ij}) = 0,} \\ &\mathbf{X_i d_{ij} + Y_i e_{ij} + Z_i f_{ij} - \beta_j (b_{xj} d_{ij} + b_{yj} e_{ij} + b_{zj} f_{ij}) = 0.} \end{aligned} \tag{34}$$

Terms $\mathbf{b_{xj}}$, $\mathbf{b_{yj}}$, and $\mathbf{b_{zj}}$ are the previously computed initial base components of image **j** with respect to the first image. The coefficients for a point **i** in image **j** are computed from

$$\begin{bmatrix} a_{ij} & b_{ij} & c_{ij} \\ d_{ij} & e_{ij} & f_{ij} \end{bmatrix} = \begin{bmatrix} -1 & 0 & x_{ij} \\ 0 & -1 & y_{ij} \end{bmatrix} C_j^{-1} R_j^T. \tag{35}$$

Note that for the first image $\mathbf{R_1 = I}$, $\mathbf{\beta_1 = 0}$ and for the second image $\mathbf{\beta_2 = 1}$. For **p** points and **k** images there are at most **2pk** equations of type (34) and **3p+k−2** unknowns in them.

5. EXAMPLE

A simple simulated numeric example of the method is presented here. The object was a cube with dimensions 1×1×1 m^3 consisting of 27 points. Five artificial images of the size 512×512 pixels were taken from the object with the focal length of approximately 950 pixels. Three of the images had the same interior orientation, and two of them had different interior orientations. The images were clearly convergent with respect to each others, and the distance to the object varied between 2.0-3.0 metres. Gaussian noise of 0.05 pixels was added to the image coordinates.

The singular correlation was computed separately for each pair of images, and the relative and five-parametric interior orientation of the images was computed from these singular correlations. Finally the model coordinates were computed, and the difference between the obtained and the true model was computed using conventional seven-parametric absolute orientation. Only the root mean square error (RMSE) after the absolute orientation was interesting because it told the orthogonality error of the obtained model.

Here, the model was obtained in dimensions of about 380×380×380 $pixel^3$, and the expected accuracy was then 1:7600 (=0.05/380). The resulting RMSE of all three coordinates was 0.237 pixels, thus the obtained relative accuracy was about 1:1600. Note that the example was not planned to check the accuracy of the method but merely to show that the method works at all.

6. CONCLUSIONS

A method for a relative orientation of multiple images was presented. It solves also the unknown interior orientation of the images without control points or approximate orientation parameters. The reconstruction is based on pure image information only, and it produces an object model in an arbitrarily scaled but orthogonal coordinate system. The method requires at least three images taken with the same camera.

The method does not assume any nonlinear deformations in the images, like radial distortion, and they have to be corrected in advance. However, it has been shown earlier that certain nonlinear image deformations can also be corrected from the images using the image information only /Kölbl, 1972/.

It is well known that a simultaneous solution of the unknown orientation parameters is usually better than a solution in steps like the one presented here. But if the correlations from previous steps are taken into account in all levels of the computations, and the dependencies of the singular correlation matrices are also taken into account, the methods should be equally accurate. Anyway, the method can be applied to obtain approximate values for a rigorous bundle adjustment of a free photogrammetric net with additional parameters.

A possible application of the method is the 3-D reconstruction of building interiors for a building information or CAD/CAM system using a hand-held video camera.

REFERENCES

Haggrén, H., Niini, I. — Relative Orientation Using 2-D Projective Transformations. The Photogrammetric Journal of Finland, Vol. 12, No. 1, 1990, pp. 22-33.

Hartley, R. — Self-Calibration from Multiple Views with a Rotating Camera. Lecture Notes in Computer Science, Vol I, No. 800, pp. 471-478. Springer-Verlag, 1994.

Jordan, Eggert, Kneissl — Handbuch der Vermessungskunde. Band IIIa/3 Photogrammetrie, Stuttgart 1972, p. 2268.

Kölbl, O. — Selbstkalibrierung von Aufnahme-kammern. Bildmessung und Luftbildwesen 1/1972, Karlsruhe, pp. 31-37.

Maybank, S., Faugeras, O. — A Theory of Self Calibration of a Moving Camera. International Journal of Computer Vision, 8:2, pp. 123-151, 1992.

Mikhail, E. — Observations and Least Squares. Thomas Y. Cromwell Company, USA, 1976.

Niini, I. — Solution of the Interior Orientation without Control Points. The Photogrammetric Journal of Finland. Vol 13, No. 2, pp. 78-84, Espoo 1993.

Niini, I. — Relative Orientation of Multiple Images using Projective Singular Correlation. Licentiate's Thesis. Espoo, 1994.