A Least Median of Squares Registration Method

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ABSTRACT

An automatic robust registration of 3-D shapes is a valuable tool when same or different type of information are needed in the same coordinate system. So, the registration is a problem where the rigid motion and the correspondence information is unknown. Actually the main problem is to search the correspondences. After the registration it is, for example, possible to fuse information, or measure differences between data sets, or the registration builds a larger data set from small data sets. The basic Iterative Closest Point algorithm is applied with the Least Median of Squares regression. A robust regression is needed because data sets can have outliers, large differences and occlusions. The tests show that the algorithm gives much better result than the basic least squares based algorithm. The test results are given also for a more simple case where just the rigid motion between data sets is unknown. In this case the Least Median of Squares algorithm found over 99 % of the outliers.

1. INTRODUCTION

The background of our work is in the close range photogrammetric problems. Typically the measured 3-D data sets are registered to build a more comprehensive data set or the measured data is compared to the Computer Aided Design (CAD) data.

Iterative Closest Point (ICP) algorithm has been used in many research studies. The basic algorithm is in (Besl and McKay). It is quite a general purpose algorithm which can be used with many representations of data, point sets, line segment sets, triangle sets and implicit and parametric curves and surfaces. It does not need any feature extraction but it can be used with extracted features that have coordinate information. The data sets can be disordered, no topology or neighbourhood information is needed. If some topology information is available, for example the data has a regular domain (a regular grid), it can be used to speed up computations (Chen and Medioni). The basic algorithm is based on an iterative least squares matching where the observations (the corresponding points) are searched in every iteration by a nearest neighbour method with an Euclidean metric. Because the algorithm is iterative it converges to a local minimum. So, to find a global minimum, the algorithm needs a good initial value for an unknown rigid motion. The paper by (Besl and McKay) describes also many methods how to get the initial values for the registration algorithm.

The original ICP algorithm does not deal with the problem of statistical outliers or differences or occlusion between data sets. The paper by (Zhang) gives a statistical method to handle the previously mentioned cases. The method assumes that the distances between corresponding points are approximately normally distributed. The observations can be classified into inlier and outlier sets using some distance threshold. A threshold and inlier and outlier sets are defined in every iteration round. A distance threshold is computed using the mean distance and the sample deviation of distances, or the histogram of distances can be used to define a threshold. Heuristic histogram methods are needed because the distribution of distances is usually irregular and it will vary a lot between iterations. A threshold value leads to a binary weighting (weight is zero or one) of the observations. Generally, a number of weighting methods can be used, such as defined in robust statistical procedures, for example Huber's or Hampel's estimator (Huber). The basic idea is that the squared residuals are replaced by an another function of residuals. These methods usually assume normally distributed errors in the middle of the distribution and some other distribution in the tails.

Theories of robust regression (Huber) and (Rousseeuw and Leroy) are not treated here. We first want to fit a regression to the majority of the data and then to discover the outliers as those which possess large residuals from the robust solution (Rousseeuw and Leroy). After outliers are removed, the procedure is followed by an least squares regression. We show that the least median of squares method is very suitable for geometric 3-D registration problems.

2. ITERATIVE CLOSEST POINT ALGORITHM

The problem is to estimate an optimal solution for a rigid motion problem (1) between two data sets. This is solved by minimizing the mean square distance between the shapes. Because the corresponding points are not known they are searched using a nearest neighbour rule. This rule does not need the feature extraction or the feature comparison between data sets. The procedure is iterative and new corresponding points are searched in every iteration. Now we assume no statistical outliers.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + R \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \tag{1}$$

$$R = \begin{bmatrix} \cos\varphi\cos\kappa & -\cos\varphi\sin\kappa & \sin\varphi \\ \cos\omega\sin\kappa+\sin\omega\sin\varphi\cos\kappa & \cos\omega\cos\kappa-\sin\omega\sin\varphi\sin\kappa & -\sin\omega\cos\varphi \\ \sin\omega\sin\kappa-\cos\omega\sin\varphi\cos\kappa & \sin\omega\cos\kappa+\cos\omega\sin\varphi\sin\kappa & \cos\omega\cos\varphi \end{bmatrix}$$

The motion (1) parameters, three angles and shifts, are solved at the same time in an iterative process. First the model is linearised by the first order Taylor series and this linear model forms the error equations of the least squares problem. Also the direct methods (Kanatani), (Besl and Mckay) and (Zhang) where the motion problem is divided into two parts are used commonly with the ICP algorithm.

Basic ICP algorithm:

- The two data sets are called a data shape and a model shape.
- Initial values for the motion parameters.
- Iteration threshold(s).
- Iterate until convergence:

  - Apply the motion to all points in the data shape.
  - Find the closest points (the closest point for all data shape points in the model shape).
  - Compute the motion between the data shape and the model shape. Use the least squares method.

The Euclidean distance computations are needed in the closest point search. The distance between two points and the distance between a point and a line and the distance between a point and a triangle have closed form solutions. Iterative methods are needed if the distance from a point to an implicit or a parametric curve or surface is computed. The search process can be speed up using special techniques from computational geometry (Preparata and Shamos) and (Dasarathy), for example (Zhang) has used k-dimensional binary tree, k-D tree. The ICP algorithm has proved monotonic convergence to a local minimum (Besl and McKay).

The basic algorithm is not able to handle statistical outliers, differences or occlusions, actually those may spoil a least squares analysis. Heuristic rules may be used to find doubtful observations and a more robust iteratively weighted least squares analysis has been used in those cases (Zhang). A local neighbourhood information, for example a local orientation angle difference between the closest surface points, can have a threshold value. A maximum distance tolerance is another good threshold value.

3. LEAST MEDIAN OF SQUARES ALGORITHM

The Least Median of Squares (LMS) (Rousseeuw and Leroy) is a very robust regression method. It has the highest possible breakdown point of 50%. The breakdown point is the smallest fraction of contamination that cause the estimator to take on values arbitrary far from the case where outliers do not exist. A high breakdown point is not a sufficient condition for a good method but it is considered as a necessary condition for a method which does not give a arbitrarily bad result. The least squares estimator has a breakdown point of 0%. This means the least squares method is extreme sensitive to outliers.

The least median of squares estimator (2) minimizes the median of squared residuals (r_i).

$$\text{Minimize median of } r_i^2 \quad i=1,\ldots,n \tag{2}$$

The least squares method minimizes the sum of the squared residuals, so the sum is replaced by the median. The algorithm we have used to compute the LMS regression coefficients is (Rousseeuw and Leroy):

LMS Algorithm:

Repeat m times:

- Draw a (random) subsample of p different observations.
- Estimate the regression coefficients from that subsample. The solution is called a trial estimate.
- Determine the median of squared residuals using the trial estimate. Residuals are computed from the whole data set.
- If the median is less than the previously saved median:
  - Save the median and the trial estimate.

So, the algorithm clearly will minimize the median of the squared residuals from the m subsamples. A problem is how many subsamples should we consider? If all possible cases are tried, there will be C_n^p different subsample combinations. A subsample has p observations from the whole set of n observations. C_n^p increases very fast with n and p. In order to reduce the possibly extreme amount of computations we accept a probability of error in recovering the correct regression coefficients. In practice, only one good subsample is needed. So, we take so many random subsamples that the probability that at least one subsample is good is close to 1. If n/p is large the probability, P(g), that at least one subsample is good is

$$P(g) = 1 - (1 - (1 - \varepsilon)^p)^m \tag{3}$$

ε is the fraction of bad observations. Now one can compute the number of subsamples, m, for given values of P(g), p and ε. The minimum number for p is the number of regression unknowns and if this number is large the number of required subsamples becomes so large that the computation time is impractical.

The classification of original observations into inlier and outlier sets is necessary if the least squares regression coefficients are wanted. The so called initial scale estimate of the LMS regression is

$$s^0 = 1.4826(1+5/(n-p))\sqrt{\text{median of } r_i^2} \qquad i=1,\ldots,n \tag{4}$$

The constant 1.4826 is the correction factor for the LMS scale estimate if the residuals are assumed to have the normal distribution. If residuals are assumed to be from some other distribution the correction factor should be changed. 1+5(n-p) is the finite sample size correction factor. The following rule gives the inliers

$$\text{if } \left|\frac{r_i}{s^0}\right| \le t, \text{ then i:th observation is inlier.} \tag{5}$$

A usual choice for an arbitrary threshold t is 2.5. It is quite reasonable because in the normal distribution case there will be very few residuals larger than $2.5s^0$.

It is also possible to take subsamples as long as some predefined LMS scale estimate is reached. In this case we accept an error caused by the difference between the real and the predefined LMS scale. Also an upper bound for the number of subsamples is needed because the algorithm will never stop if the predefined LMS scale is too small. A practical upper bound is computed from the equation (3).

4. LEAST MEDIAN OF SQUARES ICP ALGORITHM

The registration algorithm is:
- Initial values for the motion parameters.
- Iteration threshold(s).
- Repeat m times:
  - Draw a (random) subsample of p different observations from the data shape.
  - Apply the ICP algorithm to a subsample and the model shape. The result is the trial estimate.
  - Determine the median of squared residuals using the trial estimate. (Transform the data shape using the trial estimate and compute the corresponding point for all data shape points).
  - If the median is less than a previously saved median:
    - Save the median and the trial estimate.
- If more precise registration (Least Squares) is needed.
  - Compute the initial LMS scale estimate (4).
  - Classify both shapes into inlier and outlier sets (5).
  - Apply the ICP algorithm to inliers of both shapes.

Remark. In the classification step the corresponding points of inlier data shape points are the inliers of the model shape points. This is a reasonable choice because the main problem is in fact the correspondence problem. We do not compute another registration by taking the subsamples from the model shape.

The number of observations in a subsample is usually the number of regression unknowns. In the registration problem we have used at least three points which makes nine observations. In some cases three points can fit into any part of the model shape but the median of squared residuals have the minimum in the optimal LMS registration case. A random subsample represents better the data shape if it contains more than three good points. The larger the subsample, the more subsamples are needed to get the same probability that at least one subsample is good (3). Each subsample registration problem converges monotonically to a local minimum.

In (Masuda and Yokoya) has been investigated a LMS-ICP algorithm in the range image registration. They do not use the least squares step after the LMS-ICP algorithm. Because the data is in regular domain, a good approximation for a closest point pair is a pixel pair having the same row and column coordinates.

5. EXPERIMENTAL RESULTS

The first experiment type solves just the rigid motion between two sets. The corresponding points are known. We wanted to compare the precision and the accuracy of the LMS-ICP registration and the LMS rigid motion problems. We have the results from the 20 point and

Std	LMS	LS
σ_1	1.01	17.50
$\sigma_{.95}$	0.98	17.49
$\sigma_{.90}$	0.97	17.50

Table 1. Standard deviations. Rigid motion, 20 points

Std	LMS	LS
σ_1	1.01	17.62
$\sigma_{.95}$	1.00	17.60
$\sigma_{.90}$	1.00	17.60

Table 2. Standard deviations. Rigid motion, 212 points

the 212 point cases. So, we can compare also the effect of the redundancy. The coordinates of the model shape are random points from uniform distribution [-500, 500]. A random rigid transformation is also from the uniform distribution, shifts [-500, 500] and angles [-360°, 360°]. This random motion creates the data shape from the model shape. The normal distributed random noise, N(0,1), is added to the data shape points. Also an extra noise (outliers) is added to the data shape points from the uniform distribution [-50, 50]. Because outliers are from the uniform distribution it is more difficult to find them. Outlier noise is added to a random fraction, [0.30, 0.45], of the data shape points. Both the 20 and the 212 point cases were repeated 1000 times. The LMS solution has the least squares step after the subsampling process. The LMS result is compared to the least squares result. The Tables 1. and 2. show the estimated standard deviations. σ_1 is the mean of standard deviations from the 1000 cases. $\sigma_{.95}$ and $\sigma_{.90}$ are the means of the standard deviations from the best 950 and 900 LMS cases (the smaller the standard deviation the better the result). In each LMS case we took a subsample of three points 120 times. The result is that the standard deviation of noise was correctly estimated by the LMS method.

The real errors were measured by the root mean square error (rmse) between the estimated and the known rigid motion, see Table 3. and 4. r_1 is the mean of the root mean square errors from 1000 cases. $r_{.95}$ and $r_{.90}$ are the means of the root mean square errors from the best 950 and 900 LMS cases. This sorting based on the standard deviations. The conclusions from Table 3. and 4. are: The larger redundancy gives a better accuracy and the LMS algorithm gives much better accuracy than the least squares algorithm. The initial values for the algorithms were the same for all the cases (T_x=T_y=T_z=0), (ω=φ=κ=0°).

	LMS method						Least squares method					
rmse	ω	φ	κ	T_x	T_y	T_z	ω	φ	κ	T_x	T_y	T_z
r_1	1.83	0.04	1.83	0.44	0.45	0.43	6.73	0.60	6.74	6.12	5.98	5.81
$r_{.95}$	1.87	0.04	1.87	0.43	0.44	0.42	6.82	0.60	6.83	6.12	6.00	5.76
$r_{.90}$	1.87	0.04	1.87	0.43	0.43	0.42	6.95	0.60	6.95	6.07	6.02	5.75

Table 3. Root mean square errors. Rigid motion, 20 points.

	LMS method						Least squares method					
rmse	ω	φ	κ	T_x	T_y	T_z	ω	φ	κ	T_x	T_y	T_z
r_1	0.17	0.01	0.17	0.13	0.13	0.13	2.64	0.16	2.64	1.75	1.77	1.76
$r_{.95}$	0.17	0.01	0.17	0.13	0.13	0.13	2.61	0.16	2.61	1.73	1.76	1.76
$r_{.90}$	0.17	0.01	0.17	0.13	0.13	0.12	2.67	0.16	2.67	1.71	1.74	1.75

Table 4. Root mean square errors. Rigid motion, 212 points.

The second experiment is a registration problem. The model shape is a Computer Aided Designed windshield edge. The box dimensions around the edge are approximately 800 in the x-coordinate, 1400 in the y-coordinate, and 400 in the z-coordinate. A random motion transformation, the data shape, normal distributed noise and outliers were generated like in the first experiment. A systematic deformation was also added to a random part of the data shape edge. The subsampling was done in the same way than in the first experiment and a registration problem was repeated 1000 times.

The initial values for a motion were computed by a special algorithm. The ICP registration algorithm needs good initial values. If the values from the first experiment are used the algorithm do not usually give a good result. The basic idea in the initial value problem is that the rotation matrix between the eigenvectors of the scatter matrices of the shapes (Duda and Hart) is computed. Because the sign of the eigenvectors cannot be determined this procedure produces four initial rotations (3D-case). As an initial state we use the rotation which produces the best match between the data and the model shape. Some low order central moments of a point set are the features in this matching process. The method works usually well if the eigenvalues of a scatter matrix have some level of distinctness and the global shapes of both data sets are rather similar.

The results from the registration experiment are shown in Table 5. and 6 ($\sigma_1,\sigma_{.95},\sigma_{.90}$, $r_1,r_{.95},r_{.90}$ have the same meaning as in the first experiment). The noise standard deviation is not correctly estimated by the LMS method as in the first experiment. But the LMS-ICP method produced a much better precision and accuracy than the Least Squares registration method.

The experiments show that the LMS method can tolerate large fraction of outliers well. The classification of observations to inliers and outliers depends on the arbitrary threshold value (5). In these experiments the threshold value was 2.

Std	LMS	LS
σ_1	2.01	14.76
$\sigma_{.95}$	1.59	14.74
$\sigma_{.90}$	1.23	14.71

Table 5. Standard deviations from the registration cases.

Figures 1. and 2. give an example of a ICP least squares registration and Figures 3. and 4. give the LMS-ICP registration result. We have also registered profile data produced by the light striping method (Pöntinen). The LMS-ICP algorithm has given visually satisfactory results.

	LMS-ICP method						ICP Least squares method					
rmse	ω	φ	κ	T_x	T_y	T_z	ω	φ	κ	T_x	T_y	T_z
r_1	3.57	0.26	3.57	5.69	19.57	3.68	8.17	0.41	8.17	9.28	26.83	12.63
$r_{.95}$	0.76	0.14	0.79	3.47	13.01	2.24	7.66	0.37	7.66	8.49	23.74	12.37
$r_{.90}$	0.47	0.03	0.48	0.82	2.13	1.24	7.84	0.35	7.84	8.13	21.52	12.33

Table 6. Root mean square errors from the registration cases.

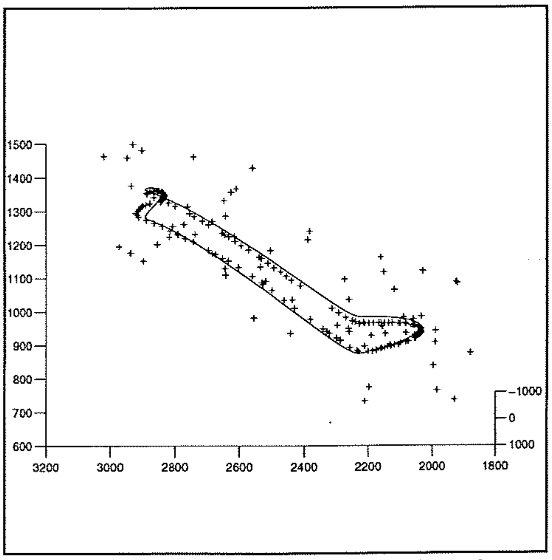

Figure 1. ICP Least Squares registration

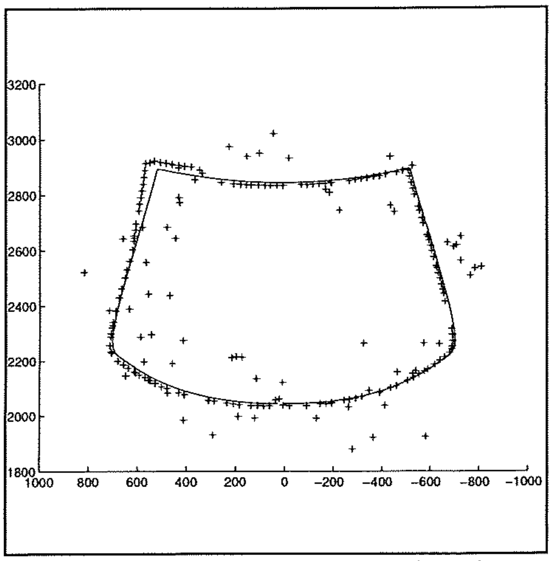

Figure 2. ICP Least Squares registration

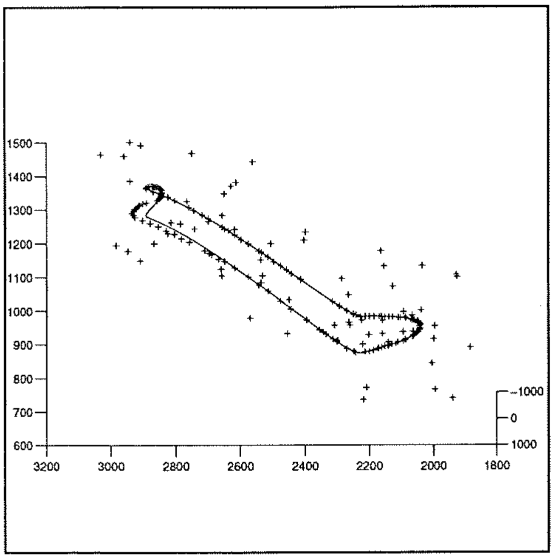

Figure 3. LMS-ICP registration

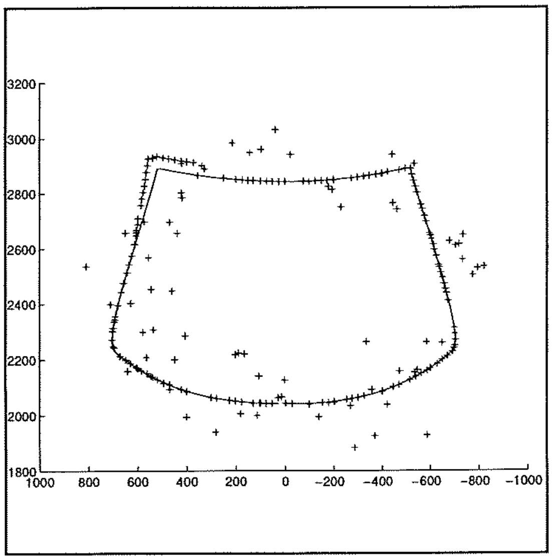

Figure 4. LMS-ICP registration

REFERENCES

Besl. P. J., McKay. N. D., 1992. A method for registration of 3-D shapes. IEEE Transactions on Pattern Analysis and Machine Intelligence 14, pp. 239-256.

Chen. Y., Medioni. G., 1992. Object Modelling By Registration of Multiple Range Images. Image and Vision Computing 10, pp. 145-155.

Dasarathy. B. V., (editor), 1991. Nearest Neighbor (NN) Norms: Nearest Neighbor Pattern Classification Techniques. IEEE Computer Society Press, Los Alamitos, California, U.S.A.

Duda. R., Hart. P., 1973. Pattern Classification and Scene Analysis. John Wiley & Sons, New York, U.S.A.

Huber. P. J., 1981. Robust Statistics. John Wiley & Sons, New York, U.S.A.

Kanatani. K, 1994. Analysis of 3-D Rotation Fitting. IEEE Transactions on Pattern Analysis and Machine Intelligence 16, pp. 543-549.

Masuda. T., Yokoya. M., 1995. A Robust method for Registration and Segmentation of Multiple Range Images. Computer Vision and Image Understanding 61, pp. 295-307.

Preparata. F. P., Shamos. M. I., 1985. Computational Geometry, An Introduction. Springer-Verlag, New York, U.S.A.

Pöntinen. P., 1996, Light striping using projective transformations. Elsewhere in this number of the Journal.

Rousseeuw. P. J., Leroy. A. M., 1987. Robust Regression and Outlier Detection. John Wiley & Sons, New York, U.S.A.

Zhang. Z., 1994. Iterative point matching for registration of free-form curves and surfaces. International Journal of Computer Vision 13, pp. 119-152.