Light striping using projective transformations

Petteri Pöntinen
Helsinki University of Technology
Institute of Photogrammetry and Remote Sensing
Otakaari 1 FIN-02150 Espoo
Finland

Abstract

Many photogrammetrical measuring systems utilize structured light. One of the simpliest cases of structured light is light striping. This article demonstrates a principle how a light striping system can be realized using projective transformations. Instead of solving the orientations of the camera and the projecting device we solve the projective transformation from the image plane to the plane defined by the projecting device.

1. Introduction

Light striping is a relatively simple technique used to derive the three dimensional shape of an object [Ballard et al.]. Systems based on light striping consist of two parts; a camera and projecting device (see Fig. 1 a). A stripe is projected on the surface of the object and registered by a camera. If the orientation of the camera and the light plane defined by the projecting device are known, the "real" three dimensional shapes of the stripes can be calculated. The projection center and all the image points belonging to the stripe define lines in a three dimensional space. Using these lines all the recorded stripes can be projected back to the light plane (see Fig. 1 b). As mentioned earlier this method requires the orientations of the camera and the light plane to be solved. But there is also an alternative way to do light striping based on projective transformations.

2. Mathematical background

Figure 2 illustrates a projection of the point P to an image plane. First we need to choose an arbitrary object coordinate system (dashed line). The image plane is defined by the vector $\bar{n}$ (the normal vector of the image plane) and any point belonging to the plane. Let's choose this point to be defined by vector $\bar{r}_0$ (vector from the origin of the object coordinate system to the origin of the image coordinate system). The vectors $\bar{x}$ and $\bar{y}$ define the image coordinate axises. For vectors $\bar{x}$,$\bar{y}$ and $\bar{n}$ holds

$$|\bar{n}| = |\bar{x}| = |\bar{y}| = 1 \qquad (1)$$

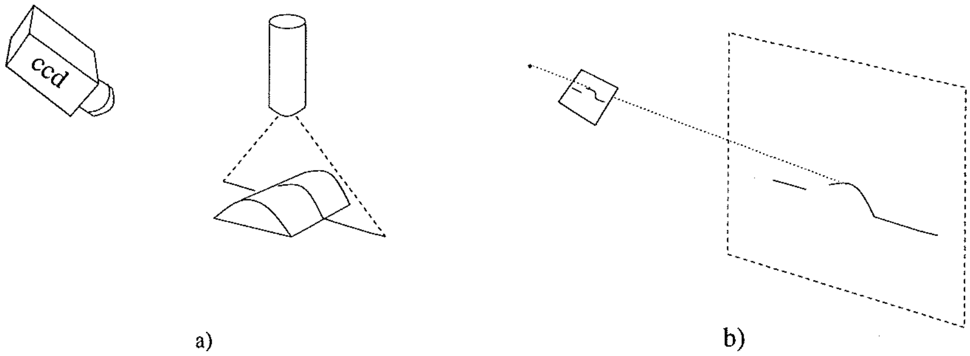

Figure 1: Principle of light striping.

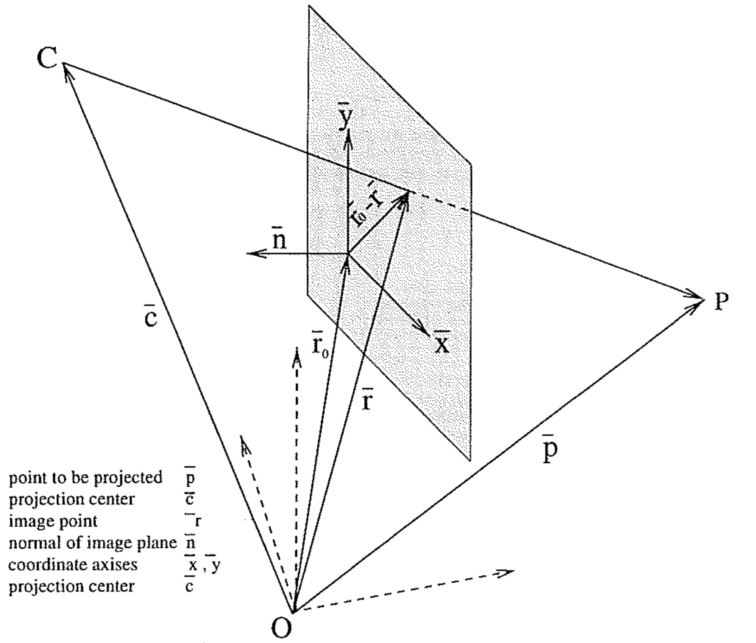

Figure 2: Projection of a point to a plane.

and of course $\bar{x}$ and $\bar{y}$ are perpendicular, which can be expressed

$$\bar{x} \cdot \bar{y} = 0. \tag{2}$$

Any point $\bar{h}$ on the image plane must satisfy the equation

$$\bar{n} \cdot (\bar{h} - \bar{r}_0) = 0. \tag{3}$$

Vector $\bar{r}$ can be formlated as

$$\bar{r} = \bar{c} + s(\bar{p} - \bar{c}), \tag{4}$$

where s is an unknown coefficient. But because $\bar{r}$ points to the image plane, $\bar{n} \cdot (\bar{r} - \bar{r}_0) = \bar{n}(\bar{c} + s(\bar{p} - \bar{c}) - \bar{r}_0) = \bar{n} \cdot (\bar{c} - \bar{r}_0 + s(\bar{p} - \bar{c})) = 0$ and s can be solved,

$$s = \frac{\bar{n} \cdot (\bar{c} - \bar{r}_0)}{\bar{n} \cdot (\bar{c} - \bar{p})}. \tag{5}$$

The image x-coordinate x' can be calculated by projecting the vector $\bar{r} - \bar{r}_0$ to the vector $\bar{x}$

$$x' = \bar{x} \cdot (\bar{r} - \bar{r}_0) \tag{6}$$

and using equation (4) we get

$$x' = \bar{x} \cdot [\bar{c} + s(\bar{p} - \bar{c}) - \bar{r}_0] = \bar{x} \cdot ((\bar{c} - \bar{r}_0) + s(\bar{p} - \bar{c})). \tag{7}$$

Substituting (5) to (7) we get

$$\begin{aligned} x' &= \bar{x} \cdot ((\bar{c} - \bar{r}_0) + \frac{\bar{n} \cdot (\bar{c} - \bar{r}_0)}{\bar{n} \cdot (\bar{c} - \bar{p})}(\bar{c} - \bar{p})) \\ &= \bar{x} \cdot \frac{[\bar{n} \cdot (\bar{c} - \bar{p})](\bar{c} - \bar{r}_0) - [\bar{n} \cdot (\bar{c} - \bar{r}_0)](\bar{c} - \bar{p})}{\bar{n} \cdot (\bar{c} - \bar{p})}. \end{aligned} \tag{8}$$

Using rule $(a \cdot c)b - (a \cdot b)c = a \times (b \times c)$ we get

$$x' = \bar{x} \cdot \frac{\bar{n} \times [(\bar{c} - \bar{r}_0) \times (\bar{c} - \bar{p})]}{\bar{n} \cdot (\bar{c} - \bar{p})} \tag{9}$$

and rule $a \cdot (b \times c) = (a \times b) \cdot c$ gives us

$$x' = \frac{(\bar{x} \times \bar{n}) \cdot [(\bar{c} - \bar{r}_o) \times (\bar{c} - \bar{p})]}{\bar{n} \cdot (\bar{c} - \bar{p})}. \tag{10}$$

For our right hand coordinate system holds $\bar{x} \times \bar{n} = -\bar{y}$ and we get finally

$$x' = -\frac{\bar{y} \cdot [(\bar{c} - \bar{r}_0) \times (\bar{c} - \bar{p})]}{\bar{n} \cdot (\bar{c} - \bar{p})}. \tag{11}$$

The y'-coordinate we get using the same strategy

$$y' = \bar{y} \cdot (\bar{r} - \bar{r}_0). \tag{12}$$

Comparing equations (6) and (12) we can immediately write

$$y' = \frac{(\bar{y} \times \bar{n}) \cdot [(\bar{c} - \bar{r}_o) \times (\bar{c} - \bar{p})]}{\bar{n} \cdot (\bar{c} - \bar{p})}. \tag{13}$$

In a right hand system $\bar{y} \times \bar{n} = \bar{x}$ and this leads to

$$y' = \frac{\bar{x} \cdot [(\bar{c} - \bar{r}_0) \times (\bar{c} - \bar{p})]}{\bar{n} \cdot (\bar{c} - \bar{p})}. \tag{14}$$

Substituting

$$\bar{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}, \bar{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}, \bar{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \bar{r}_0 = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \text{ and } \bar{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{15}$$

to equation (11) and grouping the terms gives us

$$\begin{aligned} x' = {} & \frac{(-y_2c_3 + y_2r_3 + y_3c_2 - y_3r_2)x}{(-n_1)x + (-n_2)y + (-n_3)z + (n_1c_1 + n_2c_2 + n_3c_3)} + \\ & \frac{(y_1c_3 - y_1r_3 - y_3c_1 + y_3r_1)y}{(-n_1)x + (-n_2)y + (-n_3)z + (n_1c_1 + n_2c_2 + n_3c_3)} + \\ & \frac{(-c_2y_1 + r_2y_1 + c_1y_2 - r_1y_2)z}{(-n_1)x + (-n_2)y + (-n_3)z + (n_1c_1 + n_2c_2 + n_3c_3)} + \\ & \frac{(-r_2c_3y_1 + c_2r_3y_1 - c_1r_3y_2 + y_2r_1c_3 - y_3r_1c_2 + y_3c_1r_2)}{(-n_1)x + (-n_2)y + (-n_3)z + (n_1c_1 + n_2c_2 + n_3c_3)}. \end{aligned} \tag{16}$$

It's easy to see that also for y' we get the same kind of representation. By renaming terms we can write

$$x' = \frac{a_1x + b_1y + c_1z + d_1}{a_0x + b_0y + c_0z + d_0}, \tag{17}$$

$$y' = \frac{a_2x + b_2y + c_2z + d_2}{a_0x + b_0y + c_0z + d_0}. \tag{18}$$

These are the familiar DLT (Direct Linear Transformation) - equations. They describe the connection between a three dimensional object point and a two dimensional image point. Let's think that instead of one we have two image planes intersecting the same central projection (see fig. 3 a). This time we choose the object coordinate system so that it is equal to the image coordinate system of the second image (see fig. 3 b). The DLT-equations (17) and (18) are still valid. Because of the collinearity, both p and p' are projected to the point p". It means that instead of using the point $(x\ y\ z)^T$ we can use point $(x'\ y'\ 0)^T$ in order to calculate x'' and y''. So we get

$$x'' = \frac{p_1x' + p_2y' + p_3 \cdot 0 + p_4}{p_9x' + p_{10}y' + p_{11} \cdot 0 + p_{12}} = \frac{p_1x' + p_2y' + p_4}{p_9x' + p_{10}y' + p_{12}}, \tag{19}$$

$$y'' = \frac{p_5x' + p_6y' + p_7 \cdot 0 + p_8}{p_9x' + p_{10}y' + p_{11} \cdot 0 + p_{12}} = \frac{p_5x' + p_6y' + p_8}{p_9x' + p_{10}y' + p_12}. \tag{20}$$

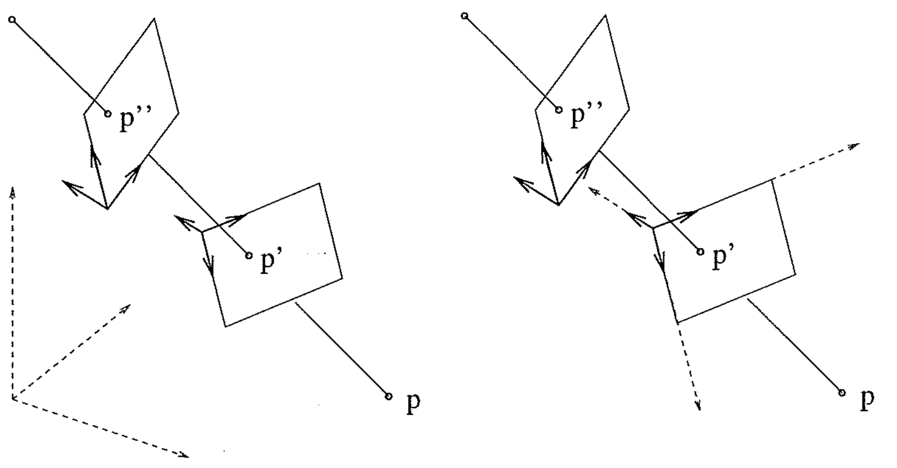

Figure 3: a) Two planes intersecting the same projection. b) Moved object coordinate system.

Dividing the nominators and the denominators of equations (19) and (20) by p_{12} gives

$$x'' = \frac{a_1x' + a_2y' + a_3}{a_7x' + a_8y' + 1}, \tag{21}$$

$$y'' = \frac{a_4x' + a_5y' + a_6}{a_7x' + a_8y' + 1}. \tag{22}$$

Equations (21) and (22) are the projective formulas from a plane to an other plane. The light striping principle presented in the next section is based on these formulas.

3. Projective solution for light striping

There are eight unknown parameters in equations (21) and (22). So we need a set of at least eight equations (which means four point pairs) to solve these unknowns. After solving the parameters we can calculate for all (x', y') points the corresponding (x'', y'') points. Also in a light striping system we have two planes, the image plane inside the camera and the light plane. The transformation parameters between these two planes are solvable by using some simple (but not always easy) arrangements. We can for example put a grid coplanar to the light plane and take a picture of it. After measuring the coordinates of the grid points on the image and on the grid, transformation parameters from image coordinates to grid coordinates can be calculated. Because the grid is coplanar to the light plane, we actually have the transformation from the image plane to the light plane. And instead of grid any object which is known by the shape (see fig. 4) can be used. Stripe on the image is a perspective view of the "true" shape. In order to get the true shape we need to project the stripe to the light plane using equations (21) and (22).

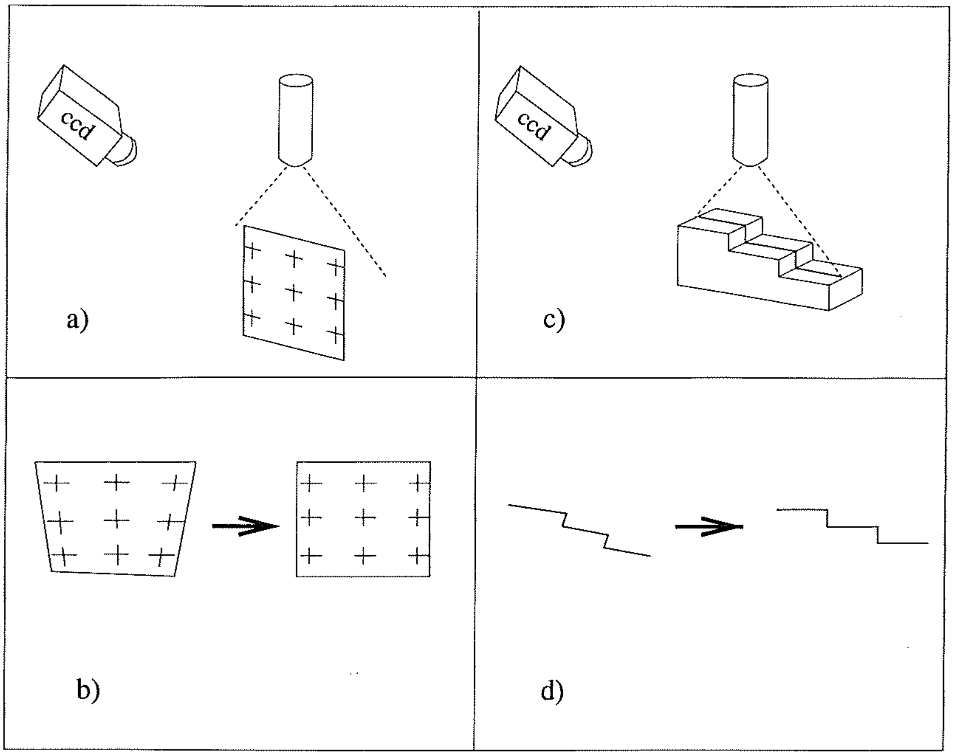

Figure 4: Two arrangements for solving the transformations. In a) coplanar grid and in c) an object which cross section is known is used. In b) and d) are the transformations to be solved respectively.

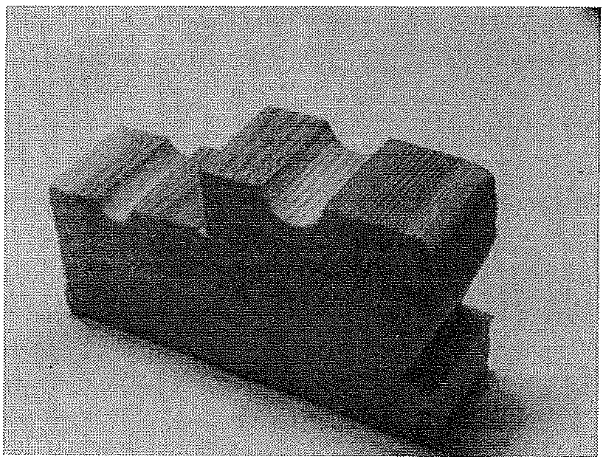

Figure 5: Object to be measured.

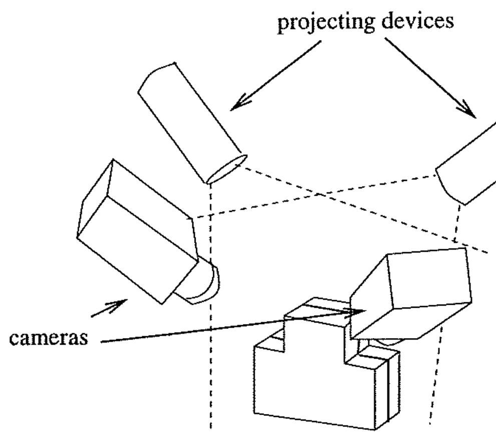

Figure 6: Set up in the example case.

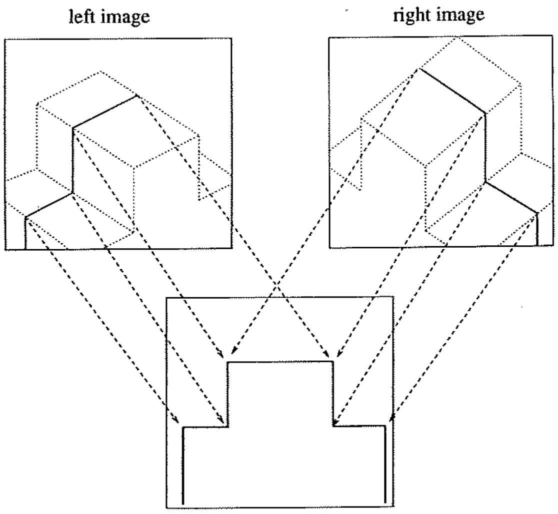

Figure 7: Two transformations to be solved.

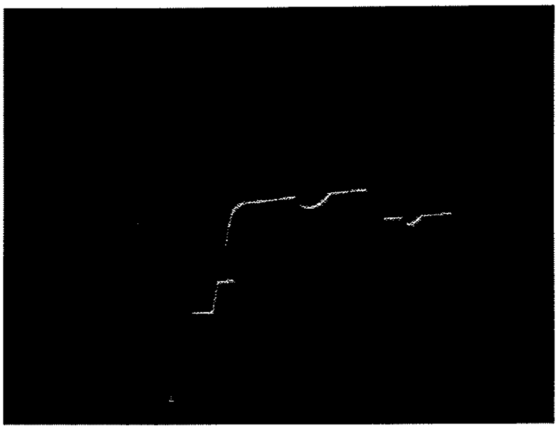

Figure 8: Left image.

Figure 9: Right image.

4. A brief example

Let's think that we want to measure a piece of wood shown in fig. 5. The first thing to do is to set up the measuring system. We decide to use two cameras and two projecting devices. We choose the camera positions so that both sides of the object will be seen. Projecting devices are located so that the two light planes are coplanar and the stripe reaches both sides of the object (see fig. 6). After these preparations the transformation parameters need to be solved. For that purpose an object, whose cross section is known, is used (see fig. 7). Because we use two cameras, we need to solve two sets of parameters. First we measure the coordinates of the break points of the stripe on the images. After that we "give" the coordinates to the corresponding points on the light plane using the fact that the cross section of the object is known. Using these point pairs we solve the unknown parameters in equations (21) and (22). After that the system is ready for use. So we place the object to

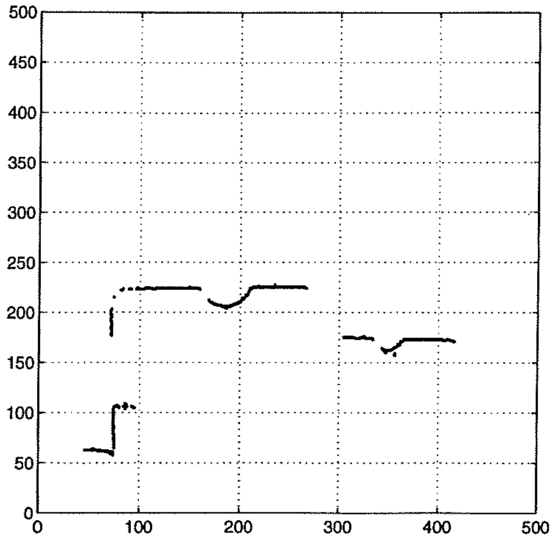

Figure 10: Transformed points from the left image.

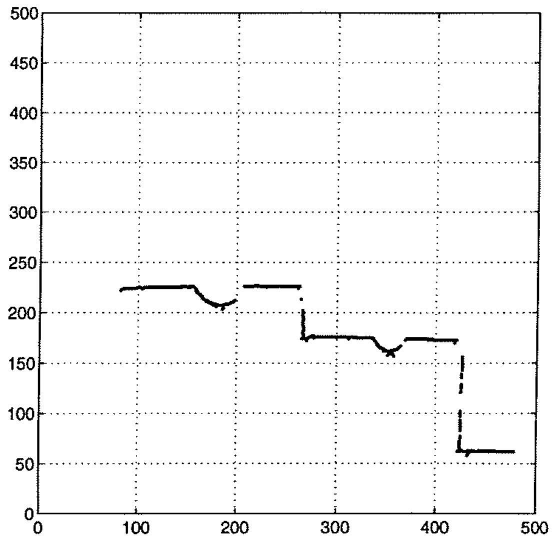

Figure 11: Transformed points from the right image.

be measured in front of the cameras and grab the images (see fig. 8 and 9). Then we search the points belonging to the stripe on the images. All found points will be transformed with calculated parameters (see fig. 10 and 11). After that the object is moved a known distance to a known direction, new images are taken, the stripe points are searched and transformed. These steps will be repeated so many times that the whole object is scanned. Because we know the movements of the object during the measurement, we can put all the transformed points in one coordinate system. As a result we have a three dimensional point cloud (see fig. 13).

5. Some remarks

If we have some previous knowledge about the accuracy of the measured image coordinates the accuracy of the system, in general, can be calculated using the error propagation law. We have to derive the accuracy of the transformation parameters and the transformed coordinates. The accuracy of a light striping system is strongly related to the measuring geometry, i.e. the positioning of the camera and the projecting device. It is quite clear without any proof that the best accuracy will be achieved using a geometry where the optical axis of the camera is perpendicular to the light plane (see fig. 14). Unfortunately with this kind of geometry the stripe will easily be occluded. An other factor which affects to the accuracy is the lens distortion. The traditional way to compensate lens distortion is the camera calibration. But it can also be partially eliminated by using projective transformations. In that case we don't have just one transformation for the whole image. The image is transformed in small pieces and each piece has its own transformation parameters. The principle is illustrated in fig. 15.

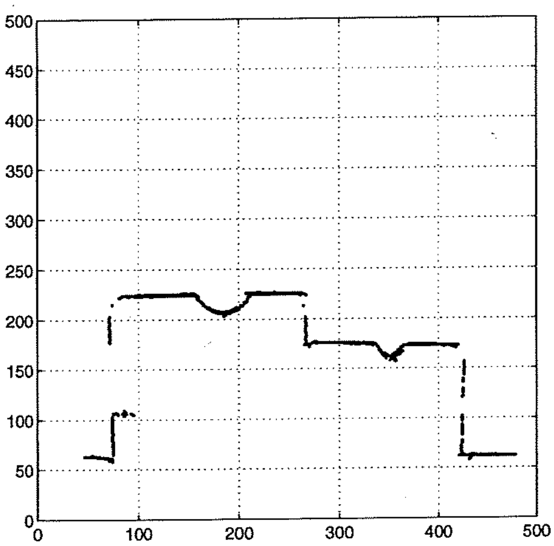

Figure 12: Transformed points from left and righ image.

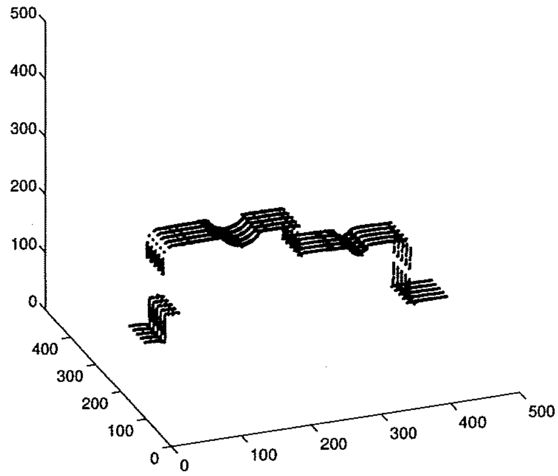

Figure 13: All captured stripes after transformation.

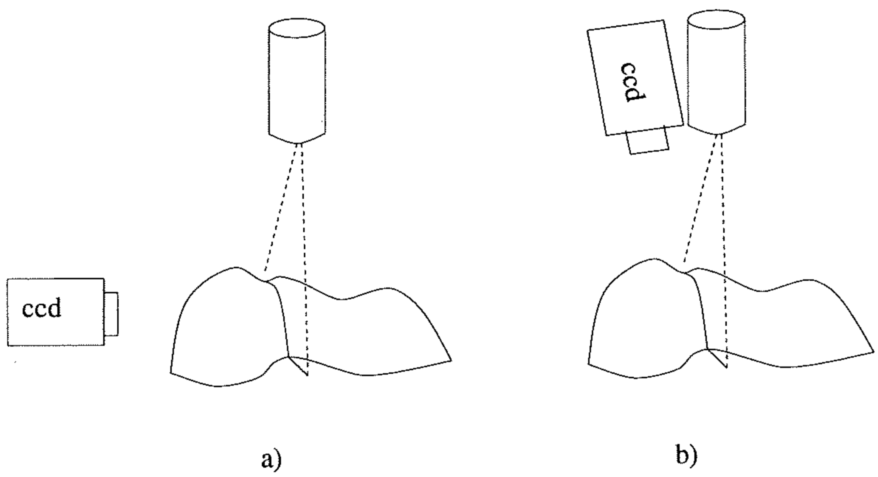

Figure 14: In a) good geometry for accuracy and bad geometry for the visibility of the stripe. In b) is the opposite.

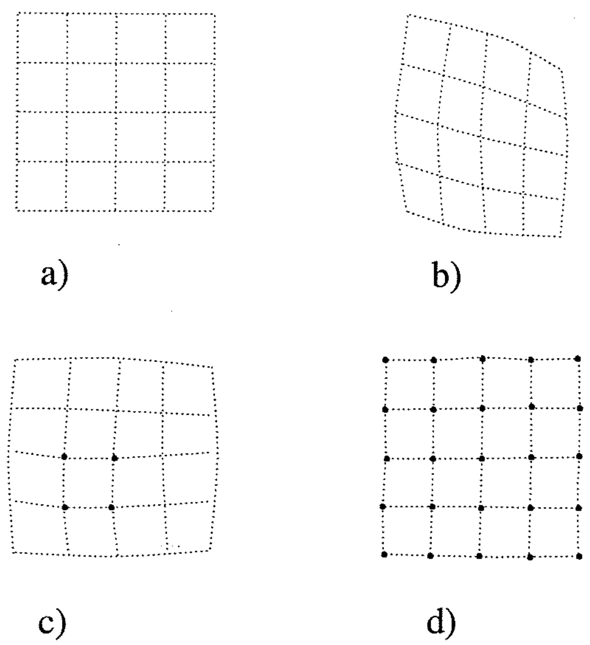

Figure 15: A partial elimination of the lens distortions. In a) is the original regular grid, in b) a perspective projection of it distorted by lens, in c) image transformed using four points and in d) using all 25 points (image is transformed in 16 different pieces).

References

[Ballard et al.] **Ballard, Dana H.; Brown, Christopher M.**, *Computer vision.* Prentice-Hall, Inc. 1982.

[Kivelä] **Kivelä, Simo K.**, *Perspektiivioppi ja aksonometria.* Otapaino 1986.

[Shirai] **Shirai, Yoshiaki**, *Three-Dimensional Computer Vision.*Springer-Verlag 1987.