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ON THE ATMOSPHERIC REFRACTION IN AERIAL PHOTOGRAMMETRY

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ABSTRACT

We present an alternative, and possibly more intuitive, method for deriving the equation for correcting the atmospheric effect in near-vertical aerial images. For near-vertical images a horizontally stratified atmospheric model is assumed to be sufficient because the amount of error at the image plane is relatively small. Our derivation utilises Snell's law and models the apparent uplift of a terrain point due refraction along the path from the ground object to the image. We show the approximate equivalence with older results in the literature, especially for aerial mapping flight heights under 7 km.

1. INTRODUCTION

The atmosphere is not homogeneous, but consists of several layers with different densities. According to Snell's law, a light ray changes its direction when passing through a boundary between two layers of different densities. This phenomenon has the unwanted effect that if an observation vector, established by an image observation, is traced, it does not lead exactly to the correct location on the target, but one that is slightly off. In other words, images are distorted because of atmospheric refraction. This is especially significant with longer distances. Awareness of the existence of atmospheric refraction long predates the invention of photography. According to Lehn and van der Werf (2005), the oldest texts on atmospheric refraction can be traced to Pliny the Elder who lived in the first century A.D., however, the discovery of the phenomenon happened most likely in the second century B.C.

In photogrammetry, an atmospheric refraction correction has traditionally been applied to aerial images. This feature is implemented in all major photogrammetric software. In practice, all image points are displaced radially outward with respect to the nadir point of an image. However, as simulations in Kyle et al. (2016) revealed, the effect should be considered also in terrestrial photogrammetry for distances longer than 10 m when sub-millimetre accuracies are required. In this paper, we focus on near-vertical aerial images.

Atmospheric refraction correction models can be divided into plane stratified and spherically stratified models. It is usually considered that plane stratified models are sufficient for vertical photography at aircraft altitudes (Gyer, 1996). However, for oblique images and images taken from high altitudes, spherically stratified models are more accurate. Plane stratified models become inaccurate when rays intersect the vertical under an angle exceeding 60 degrees.

In this paper, we present an alternative derivation of a plane stratified atmospheric refraction model for vertical aerial photography. Our model results are compared to corresponding equations from the literature. We believe that for many readers this derivation will be more intuitive than the older ones, and may serve, e.g, educational purposes. The final equation does not improve existing models, but gives an approximately similar result.

2. REFERENCE ATMOSPHERIC CORRECTION MODELS

As a reference, we have selected two models from the literature to compare with ours. The first is the commonly applied atmospheric refraction correction model, given, e.g., in Thompson et al. (1966) and Kraus (2007). In this model, the correction of the image point is given by the equation

$$\delta r_{\text{atm}} = K\left(r + \frac{r^3}{c^2}\right) = \frac{r}{\cos^2\alpha}K \tag{1}$$

with

$$K = \frac{0.00241}{Z_0}\left(\frac{Z_0^2}{Z_0^2 - 6Z_0 + 250} - \frac{Z^2}{Z^2 - 6Z + 250}\right). \tag{2}$$

Z_0 refers to the absolute flying height and Z to the ground elevation above sea level. Both variables must be in units of kilometres, while K is dimensionless. The variable c denotes the camera constant, i.e., the calibrated focal length. The angle α is defined as $\tan\alpha = {}^{r}\!/_{c}$. The variable r is actually the distance, in the image, between the nadir point and the image point. However, because this equation is an approximation for nadir aerial images, it can be assumed that the nadir point and the principal point are close to each other. Especially, in aerial triangulation this assumption is needed because the nadir point is not known before solving the exterior orientations of the images. The above expression for K is tailored for panchromatic wavelengths in the standard atmosphere assuming the ARDC model (Minzner et al., 1959). However, Bertram (1969) actually warns that equation 2 is accurate only for objects close to sea level, because it is based on a derivation (Bertram, 1966), which included an error revealed by Schut (1969). More accurate versions of K can be found, e.g., from Bertram (1969), Ghosh (1979), and Gyer (1996). Alternatively, Saastamoinen (1974) estimated K for flying heights below 11 km using the I.C.A.N. atmosphere (NACA (National Advisory Committee for Aeronautics), 1952) as

$$K = \left[\frac{2335}{Z_0 - Z}\left[(1-0.02257Z)^{5.256} - (1-0.02257Z_0)^{5.256}\right] - 277.0\,(1-0.02257Z)^{4.256}\right]\cdot 10^{-6} \tag{3}$$

and a simplified version based on the U.S. Standard Atmosphere (1962) for flying heights below 9 km as

$$K = \left[13\,(Z_0 - Z)\,[1 - 0.02\,(2Z_0 + Z)]\right]\cdot 10^{-6}. \tag{4}$$

As another reference, we applied the spherically stratified atmospheric refraction model by Gyer (1996). This model assumes the COESA (1976) standard atmosphere model. In practice, we utilized the numeric table provided in Gyer (1996).

3. THE “APPARENT UPLIFT” DERIVATION OF ATMOSPHERIC REFRACTION

If aerial imaging is done from a significant height above the ground, atmospheric refraction needs to be modelled. For vertical imaging, the correction model used will be symmetric around the vertical axis: a standard atmosphere model may be used which will be quite good enough in view of the smallness of this correction.

The refractive index n of air can be obtained from any of a number of models called “standard atmospheres”. For our purpose, correcting vertical photogrammetry, we may use an approximate expression.

We assume the atmosphere to be horizontally stratified, figure 2, and apply Snell’s law, figure 1. The stratification can be approximated by a sequence of discrete steps in refractive index n, see

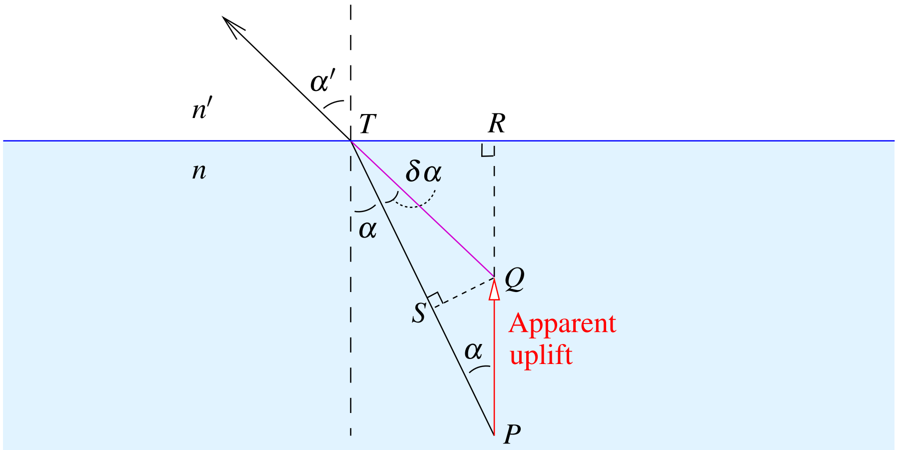

Figure 1: Snell's law: change in angle of incidence at a refractive-index interface.

figure 2. Then we have, with Snell's law for a single refractive-index interface, i.e., a step change in refractive index by $\overline{n}$, which is the ratio of the two media's diffraction indices n and n':

$$\frac{PQ}{PR} = \frac{{}^{PS}/_{\cos\alpha}}{PT\cos\alpha} = \frac{PS}{PT\cos^2\alpha} = \frac{PT - QT\cos\delta\alpha}{PT\cos^2\alpha} \approx \frac{PT - QT}{PT\cos^2\alpha}. \tag{5}$$

Here we have assumed that the change in direction $\delta\alpha$ is small, so $\cos\delta\alpha \approx 1$. Using Snell's law we see that

$$\frac{n}{n'} = \frac{\sin\alpha'}{\sin\alpha} = \frac{{}^{RT}/_{QT}}{{}^{RT}/_{PT}} = \frac{PT}{QT}, \tag{6}$$

and thus

$$\frac{PQ}{PR} = \frac{PT - ({}^{n'}/_{n})\,PT}{PT\cos^2\alpha} = \frac{1 - {}^{n'}/_{n}}{\cos^2\alpha} \implies PQ\,(\alpha) = \frac{1 - {}^{n'}/_{n}}{\cos^2\alpha}PR \approx \frac{n - n'}{\cos^2\alpha}PR = -\frac{\Delta n}{\cos^2\alpha}PR, \tag{7}$$

allowed as n is close to unity. In the small-angle limit for α we have

$$PQ\,(0) = -\Delta n\,PR \tag{8}$$

and when equations 7 and 8 are combined we get

$$PQ\,(\alpha) = \frac{PQ\,(0)}{\cos^2\alpha}. \tag{9}$$

This is the apparent uplift of a terrain point, expressed in the height difference PR between terrain and refractive-index interface, produced by a refractive-index increment at this interface. In the following we write this height difference as $PR = Z' - Z$.

Now, for a number of interfaces $k = 1, \ldots, K$, expression 8 should be summed to obtain the total uplift:

$$PQ_{\mathrm{tot}}\,(0) = -\sum_{k=1}^{K} PR_k \Delta n_k, \tag{10}$$

and in the limit for a continuous refractive-index gradient, this becomes an integral:

$$PQ_{\mathrm{tot}}\,(0) = -\int_{n(0)}^{n(Z_0)} PR\,(n)\,dn = -\int_{0}^{Z_0} PR\,(Z')\,\frac{dn\,(Z')}{dZ'}dZ'. \tag{11}$$

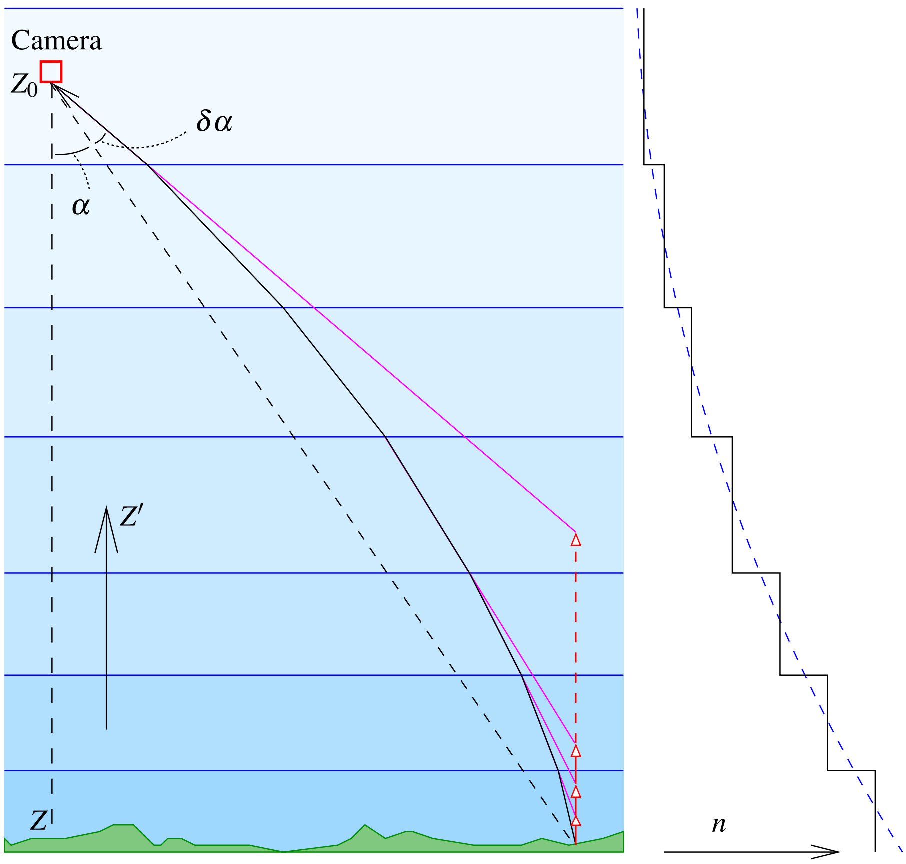

Figure 2: Atmospheric stratification and ray propagation.

Now let us assume that at a point within the atmosphere at an absolute height Z', the excess index of refraction over unity $n-1$ is an exponential function *of* Z', with a scale height τ (tau) in kilometers and a coefficient C. This is a sensible assumption if both molecular-species composition and temperature of the atmosphere, and local gravity, are constant with height. However, especially the temperature is not: its vertical gradient is strongly negative in the troposphere. The assumed simple model is

$$n\left(Z'\right)-1=C\exp\left(-\frac{Z'}{\tau}\right). \tag{12}$$

Then, the derivative of the refractive index is

$$\frac{dn\left(Z'\right)}{dZ'}=\frac{d}{dZ'}\left(n\left(Z'\right)-1\right)=-\frac{C}{\tau}\exp\left(-\frac{Z'}{\tau}\right).$$

With this, the total uplift 11 becomes

$$PQ_{\text{tot}}\left(0\right)=\int PR\frac{C}{\tau}\exp\left(-\frac{Z'}{\tau}\right)dZ'=\int\left(Z'-Z\right)\frac{C}{\tau}\exp\left(-\frac{Z'}{\tau}\right)dZ', \tag{13}$$

and this is the integral we have to evaluate all the way through the atmosphere.

$$\int \overbrace{(Z'-Z)}^{u} \overbrace{\frac{C}{\tau}\exp\left(-\frac{Z'}{\tau}\right)}^{v'} dZ' = \overbrace{(Z'-Z)}^{u} \overbrace{\left[-C\exp\left(-\frac{Z'}{\tau}\right)\right]}^{v} - \int \overbrace{1}^{u'} \overbrace{\left[-C\exp\left(-\frac{Z'}{\tau}\right)\right]}^{v} dZ' =$$
$$= -(Z'-Z)\,C\exp\left(-\frac{Z'}{\tau}\right) - \tau C\exp\left(-\frac{Z'}{\tau}\right) =$$
$$= -\left[(Z'-Z)+\tau\right]C\exp\left(-\frac{Z'}{\tau}\right) \tag{14}$$

using integration by parts, explicitized using the u, v notation. C is a still unknown constant.

Let us introduce the notation $I_Z^{Z_0}(\alpha)$ to denote this uplift integral from ground level Z to camera level Z_0. Then, integrating from sea level $Z = 0$ to outer space $Z_0 = \infty$ we obtain

$$I_0^{\infty}(0) = \int_0^{\infty} (Z'-0)\,\frac{C}{\tau}\exp\left(-\frac{Z'}{\tau}\right) dZ' = C\tau, \tag{15}$$

and we know this, the total atmospheric delay in zenith for visible light from sea level to space, to be approximately $\delta Z^0 \approx 2.41\,\mathrm{m}$ for dry air (e.g., Marini and Murray 1973).

Now the same uplift integral, not from sea level to outer space, but from terrain level Z to flight level Z_0, will be given by

$$I_Z^{Z_0}(0) = \int_Z^{Z_0} PQ(0)\,dZ' = \int_Z^{Z_0} (Z'-Z)\,\frac{C}{\tau}\exp\left(-\frac{Z'}{\tau}\right) dZ' =$$
$$= \left[-\left((Z'-Z)+\tau\right)C\exp\left(-\frac{Z'}{\tau}\right)\right]\Bigg|_Z^{Z_0} =$$
$$= \delta Z^0 \left[\exp\left(-\frac{Z}{\tau}\right) - \left(\frac{Z_0-Z}{\tau}+1\right)\exp\left(-\frac{Z_0}{\tau}\right)\right]. \tag{16}$$

This is the uplift for a vertical ray. For a tilted ray, tilt angle α, the integral becomes, with equation 9,

$$I_Z^{Z_0}(\alpha) = \int_Z^{Z_0} PQ(\alpha)\,dZ' = \frac{I_Z^{Z_0}(0)}{\cos^2\alpha}, \tag{17}$$

making the reasonable approximation of a constant α.

Furthermore, we see in figure 3, that the total deflection

$$\delta\alpha_{\mathrm{atm}} = \frac{QS}{SO} \approx \frac{QP\sin\alpha}{PO} = \frac{1}{PO}\cdot\sin\alpha\cdot QP = \frac{\cos\alpha}{Z_0-Z}\cdot\sin\alpha\cdot I_Z^{Z_0}(\alpha), \tag{18}$$

where we identify the inverse of the distance $PO = \dfrac{Z_0-Z}{\cos\alpha}$, the foreshortening $\sin\alpha$, and the uplift integral $I_Z^{Z_0}$. Thus the radially outward bending of the ray becomes, introducing again the dependence on α of the apparent uplift found above, equation 17:

$$\delta\alpha_{\mathrm{atm}} = \overbrace{\frac{\cos\alpha}{Z_0-Z}}^{\text{distance}^{-1}} \cdot \overbrace{\sin\alpha}^{\text{foreshortening}} \cdot \overbrace{\frac{\delta Z^0}{\cos^2\alpha}\left[\exp\left(-\frac{Z}{\tau}\right) - \left(\frac{Z_0-Z}{\tau}+1\right)\exp\left(-\frac{Z_0}{\tau}\right)\right]}^{I_Z^{Z_0}(\alpha)} =$$
$$= \frac{\tan\alpha}{Z_0-Z}\cdot\delta Z^0\left[\exp\left(-\frac{Z}{\tau}\right) - \left(\frac{Z_0-Z}{\tau}+1\right)\exp\left(-\frac{Z_0}{\tau}\right)\right]. \tag{19}$$

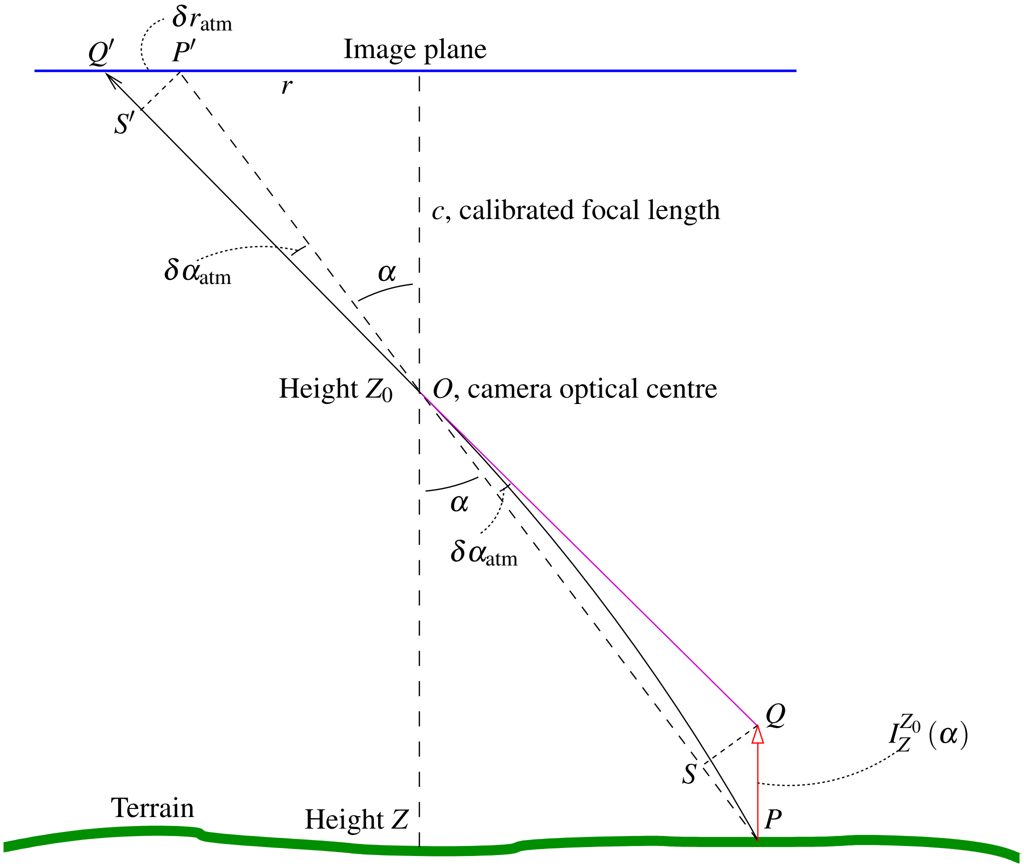

Figure 3: Geometry of ray propagation. Size of camera exaggerated.

Then, the radial offset in the image plane becomes

$$\delta r_{\mathrm{atm}} = P'Q' = \frac{P'S'}{\cos\alpha} \approx \frac{1}{\cos\alpha} OP' \delta\alpha_{\mathrm{atm}} = \frac{1}{\cos\alpha}\frac{c}{\cos\alpha}\delta\alpha_{\mathrm{atm}} = \frac{c}{\cos^2\alpha}\delta\alpha_{\mathrm{atm}}. \tag{20}$$

The variable c denotes the camera constant, i.e., the calibrated focal length. So, with the distance from the image centre $r = c\tan\alpha$,

$$\begin{aligned}\delta r_{\mathrm{atm}} &= \frac{c}{\cos^2\alpha}\delta\alpha_{\mathrm{atm}} = \frac{c\tan\alpha}{\cos^2\alpha}\cdot\frac{\delta Z^0}{Z-Z_0}\left[\exp\left(-\frac{Z}{\tau}\right) - \left(\frac{Z_0-Z}{\tau}+1\right)\exp\left(-\frac{Z_0}{\tau}\right)\right] = \\ &= \frac{r}{\cos^2\alpha}\cdot\frac{\delta Z^0}{Z_0-Z}\left[\exp\left(-\frac{Z}{\tau}\right) - \left(\frac{Z_0-Z}{\tau}+1\right)\exp\left(-\frac{Z_0}{\tau}\right)\right].\end{aligned} \tag{21}$$

Here we may use, e.g., $\tau = 8.47\,\mathrm{km}$ — roughly correct for sea-level conditions (Brekke, 2013) — and $\delta Z^0 = 2.41\,\mathrm{m}$. These values may be adjusted to better correspond to known local conditions during imaging.

We may also start from the equation for the refractive index at sea level, e.g., Rüeger (1990):

$$N_L = N_0(\lambda)\frac{273.15\,\mathrm{K}}{T}\frac{p}{1013.25\,\mathrm{hPa}} - \frac{11.27\,\mathrm{K/hPa}}{T}e,$$

with p atmospheric pressure, T absolute temperature, and e water-vapour partial pressure. Here,

$$N_0 = 287.6155 + \frac{4.8866\,\mu\mathrm{m}^2}{\lambda^2} + \frac{0.0680\,\mu\mathrm{m}^4}{\lambda^4},$$

with λ the wavelength. For λ in the range $390-1400\,\mathrm{nm}$ (visual and near infrared) we obtain $N_0(\lambda)$ in the range $290-320$. For $N_0 = 300$ and $T = 288\,\mathrm{K}$ this yields (dry air, $e = 0$) $N_L = 285$, i.e., $C = n-1 = 10^{-6}N_L = 285\cdot 10^{-6}$, resulting in $\delta Z^0 = C\tau = 2.41\,\mathrm{m}$ for an assumed scale height of $8.47\,\mathrm{km}$.

It is not clear from looking at it, but the whole right-hand side in equation 21 is to first order (i.e., for small values of $Z_0 - Z$) linear in $Z_0 - Z$ and vanishes asymptotically in the limit $Z_0 - Z \downarrow 0$. This is most easily seen by noting that the expression in square brackets, and its first derivative, vanish at $Z_0 = Z$, making this expression a to first order quadratic expression in $Z_0 - Z$ in this neighbourhood.

4. DISCUSSION

Schut (1969) presents a very similar, but simpler, geometric derivation of atmospheric refraction leading to a slightly different result. The horizontally stratified atmospheric model (Gyer, 1996) gives an essentially identical result to our derivation, however, arrived at in a very different way. The equation of Gyer's horizontally stratified model is (in our notation)

$$\begin{aligned}\delta\alpha_{\mathrm{atm}} &= \frac{\tan\alpha}{Z_0 - Z}\int_Z^{Z_0}\frac{1}{2}\frac{[n(Z') - n(Z_0)]\,[n(Z') + n(Z_0)]}{n^2(Z_0)}dZ' \approx \\ &\approx \frac{\tan\alpha}{Z_0 - Z}\int_Z^{Z_0}\left[n(Z') - n(Z_0)\right]dZ'.\end{aligned} \tag{22}$$

If we substitute here our exponential atmospheric model, equation 12, we obtain for the integral inside equation 22:

$$\begin{aligned}\int_Z^{Z_0}\left[n(Z') - n(Z_0)\right]dZ' &= \delta Z^0\left\{\left[-\exp\left(-\frac{Z'}{\tau}\right)\right]_Z^{Z_0} - \frac{Z_0 - Z}{\tau}\exp\left(-\frac{Z_0}{\tau}\right)\right\} = \\ &= \delta Z^0\left\{\exp\left(-\frac{Z}{\tau}\right) - \exp\left(-\frac{Z_0}{\tau}\right) - \frac{Z_0 - Z}{\tau}\exp\left(-\frac{Z_0}{\tau}\right)\right\} = \\ &= \delta Z^0\left[\exp\left(-\frac{Z}{\tau}\right) - \left(\frac{Z_0 - Z}{\tau} + 1\right)\exp\left(-\frac{Z_0}{\tau}\right)\right].\end{aligned} \tag{23}$$

This is identical to equation 19.

In figure 4, we compare our own simple, horizontally stratified exponential model above with the model of Gyer (1996) — which gives tabulated values that were computed based upon the Standard Atmosphere 1976 (Public Domain Aeronautical Software, 2014) and spherical geometry —, and with the equation 2 given in Kraus (2007). See table 1 for a summary.

We see that, for common aircraft heights, our model performs not a whole lot worse than the differences seen between the more sophisticated models. Undoubtedly much of the difference of our model from the older ones is due to the simplifying assumption of a constant scale height. Also note that the chromatic abberation of the atmosphere, i.e., the variation in atmospheric refraction between different parts of the spectrum, is of this order of magnitude as well. More generally, our model suffers, apart from the scale-height approximation, from the same problems as all similar models. Changes of atmospheric conditions, such as temperature, atmospheric pressure, humidity, and wavelength, change the parameters of the model. To ensure the most accurate results, these conditions should be assessed by measurement and taken into account. However, this is beyond the scope of this article.

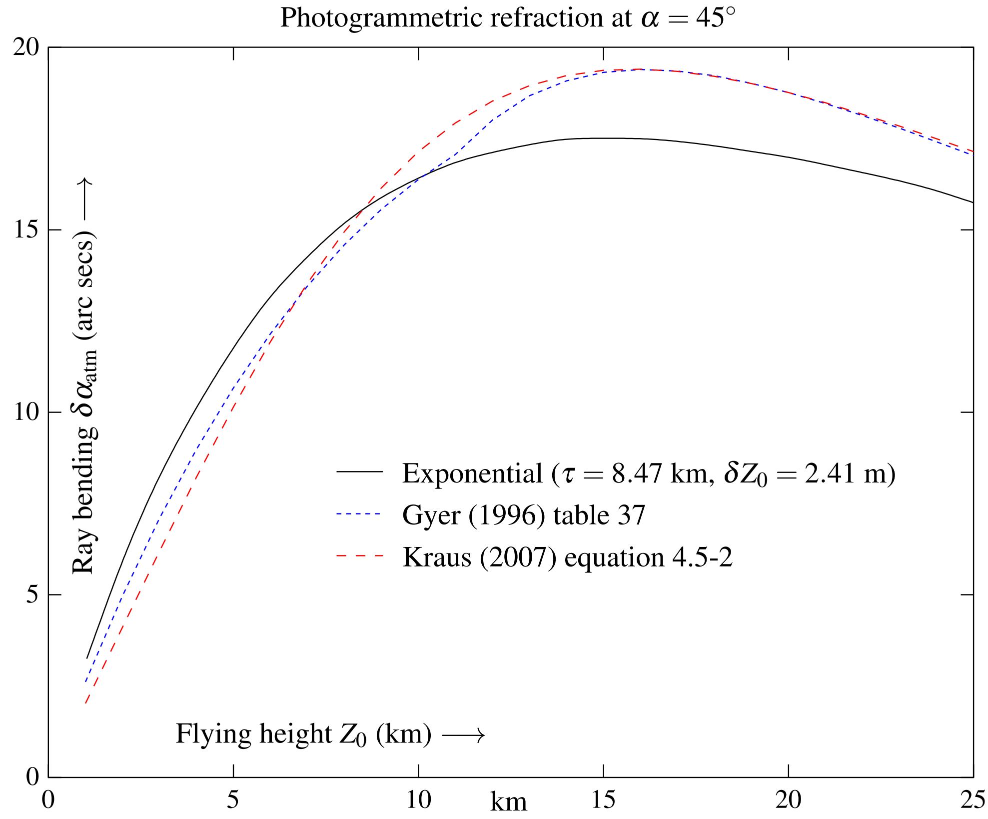

Figure 4: Comparison of atmospheric-refraction models. The terrain is assumed to be at sea level, $Z = 0$.

Table 1: Summary of atmospheric-refraction models in figure 4. Z is the assumed ground height above sea level.

Model	Wavelengths	Atmosphere	Citation	Z
Kraus (2007)	Panchromatic	ARDC 1959	Minzner et al. (1959)	0
Gyer (1996)	Visible?	Standard Atm. 1976	COESA (1976)	0
This paper	Visible + NIR	Simple exponential	-	0

5. CONCLUSIONS

The correction of atmospheric refraction is an essential part of aerial photogrammetric measurements. We have presented a geometrically illustrative method to derive the effect of atmospheric refraction for near-vertical aerial images. Our derivation assumes that the atmosphere contains horizontally stratified layers, which is not realistic enough to be applied with oblique images, with images having a wide opening angle, or with images obtained from very high altitudes. For such cases, spherically stratified models are needed.

We model the apparent uplift of a terrain point due to refraction along the path from the ground object to the image according to Snell's law. We assume that the index of refraction is an exponential function of height within the atmosphere. As a result, we obtain a model that performs approximately similarly to other models for heights up to 7 km. Our model appears to lead to the same mathematical result as the horizontally stratified refraction model in Gyer (1996) even though the derivation of that model is very different from ours. However, we believe that our derivation is more illustrative and didactically useful.

6. ACKNOWLEDGEMENTS

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